CPSC 121 (sections 203 and 204) Solutions to Sample Quiz 1

[3] 1. List and explain two advantages of two's complement over one's complement.

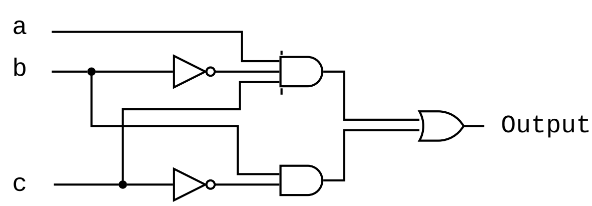
Solution: First, there is only one representation for 0 using two's complement, whereas with one's complement both 000...000 and 111...111 are used to represent 0. Second, binary addition does not always give the right answer for one's complement, whereas it does for one's complement. Both of these makes circuits that use two's complement representation easier to design than equivalent circuits that make use of one's complement.

[3] 2. Assuming integers are represented using 8-bit 2's complement, what are the binary and decimal representations of the hexadecimal value 8A?

Solution: The binary representation is 10001010. Since we are using two's complement representation, the corresponding decimal value is -128 + 8 + 2 = -118.

[7] 3. Design a circuit that takes three signals a, b and c as inputs, and outputs 1 if the string abc contains the pattern 10 but not the pattern 100, and outputs 0 otherwise. For instance, the circuit would return `true` for $a = 0, b = 1, c = 0$, but `false` for both $a = 1, b = 0, c = 0$ and $a = 0, b = 1, c = 1$.

Solution:



[12] 4. For every pair of logical expressions, either prove that they are logically equivalent, or give an example showing that they are not.

i. $\overline{x \longrightarrow y}$ and $x \wedge \overline{y}$

Solution: They are logically equivalent:

$$\begin{aligned} \overline{x \longrightarrow y} &\equiv \overline{\overline{x} \vee y} \\ &\equiv \overline{\overline{x}} \wedge \overline{y} \\ &\equiv x \wedge \overline{y} \end{aligned}$$

ii. $x \vee (y \wedge \overline{z})$ and $(x \vee y) \wedge (x \vee \overline{z}) \wedge (x \vee y \vee z)$

Solution : They are logically equivalent. This could be proved using a truth table, but here is a shorter (albeit a bit less obvious) proof. Let

$$\begin{aligned} p &: \quad x \vee (y \wedge \overline{z}) \\ q &: \quad (x \vee y) \wedge (x \vee \overline{z}) \wedge (x \vee y \vee z) \end{aligned}$$

First, observe that by the distributive law

$$p \equiv (x \vee y) \wedge (x \vee \overline{z}).$$

Hence

- if q is true, then $(x \vee y) \wedge (x \vee \overline{z})$ is also true, and hence so is p.
- if q is false, then either $(x \vee y) \wedge (x \vee \overline{z})$ is false (which means that p is false), or $x \vee y \vee z$ is false (which means that x, y and z are all false, and so p is also false).

Therefore $p \leftrightarrow q$ is a tautology. That is, the two expressions are logically equivalent.

iii. $x \leftrightarrow y$ and $(x \wedge y) \vee (x \vee \overline{y})$

Solution : These two expressions are not logically equivalent. If x is true and y is false, then $x \leftrightarrow y$ is false, but $x \vee \overline{y}$ is true, and so the right-hand side is also is true.

iv. $x \vee y \vee \overline{z}$ and $\overline{x \wedge y} \wedge (z \rightarrow (x \oplus y))$

Solution : The two expressions are not logically equivalent. If x and y are both true, then the left-hand side is clearly true, but $x \wedge y$ is true, which means that $\overline{x \wedge y}$ is false, and so the right-hand siddze is false.