1. {3 marks} Can a simple graph have 5 vertices and 12 edges? If so, draw it; if not, explain why it is not possible to have such a graph.

**ANSWER:**

In a simple graph, no pair of vertices can have more than one edge between them. In other words, there are no parallel edges.

For a simple graph, the “densest” graph we can get is one in which every vertex is connected to every other vertex. This is called a complete graph. The maximum number of edges in the complete graph containing 5 vertices is given by $K_5$: which is $C(5, 2)$ edges = “5 choose 2” edges = 10 edges. Since 12 > 10, it is not possible to have a simple graph with more than 10 edges.

2. {6 marks} Suppose that in a group of 5 people: A, B, C, D, and E, the following pairs of people are acquainted with each other.

- A and C
- A and D
- B and C
- C and D
- C and E

a) Draw a graph G to represent this situation.
b) List the vertex set, and the edge set, using set notation. In other words, show sets $V$ and $E$ for the vertices and edges, respectively, in $G = \{V, E\}$.
c) Draw an adjacency matrix for $G$.

**ANSWER:**

a) One such graph for $G$ is:

```
   A
 / \
B - C
   D
    E
```
b) For sets V and E, any order to the elements is fine. Furthermore, in edge set E, you can specify (A, C) or (C, A); they mean the same thing.

\[ V = \{A, B, C, D, E\} \]
\[ E = \{(A, C), (A, D), (B, C), (C, D), (C, E)\} \]

c) Adjacency matrix (0 = no edge; 1 = edge):

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
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</tbody>
</table>

3. {3 marks} How many more edges are there in the complete graph \( K_7 \) than in the complete graph \( K_5 \)?

**ANSWER:**

\[ C(7, 2) - C(5, 2) = 21 - 10 = 11 \]

4. {4 marks} Given a graph for a tree (with no designated root), briefly describe how a root can be chosen so that the tree has maximum height. Similarly, describe how a root can be chosen so that the tree has minimum height. (Note that path length is described as the number of edges that need to be traversed between two vertices.)

**ANSWER:**

For the maximum height, choose either end of the longest path as the root. For the minimum height, choose the vertex at the half-way point of the path.

5. {6 marks} Perform a breadth-first search of the following graph, where E is the starting node. In other words, show the output if we issue the call \( \text{BFS}(E) \). Provide two cases: (a) Use a counterclockwise ordering from the top (12 o’clock position); and (b) Use a clockwise ordering from the top.
ANSWER:

(a) When we visit adjacent nodes in a counterclockwise order from the top, the order in which we visit the nodes is:

   E, D, F, C, G, B, A

(b) When we visit adjacent nodes in a clockwise order from the top, the order in which we visit the nodes is:

   E, F, D, G, C, B, A

6. {6 marks} Perform a depth-first search of the same graph as in Question 5, but use D as the starting node. In other words, show the output if we issue the call \( \text{DFS}(D) \). Provide two cases: (a) Use a counterclockwise ordering from the top (12 o’clock position); and (b) Use a clockwise ordering from the top.

ANSWER:

(a) When we visit adjacent nodes in a counterclockwise order from the top, the order in which we visit the nodes is:

   D, E, F, C, B, A, G

(b) When we visit adjacent nodes in a clockwise order from the top, the order in which we visit the nodes is:

   D, G, C, B, A, E, F