CPSC 121,
2007/8 Winter Term 2
Sample Quiz 4

Name: ______________________________ Student ID: __________________________

Signature: ___________________________

- A total of 18 marks are available. **You may want to complete what you consider to be the easiest questions first!**
- Justify all of your answers.
- No notes or electronic equipment are allowed.
- Keep your answers short. If you run out of space for a question, you have written too much.
- The number in square brackets to the right of the question indicates the number of marks allocated to that question.
- Good luck!

Question	Marks	Out of
1		6
2		4
3		8
Total		18

UNIVERSITY REGULATIONS:

- Each candidate should be prepared to produce, upon request, his/her university-issued ID.
- No candidate shall be permitted to enter the examination room after the expiration of one half hour or to leave during the first half hour of the examination.
- CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
    1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
    2. Speaking or communicating with other candidates.
    3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

1. **[6]** Design a DFA over the input alphabet $\Sigma = \{A, B, C, \ldots, Z\}$ that accepts all strings that end in LY but don't start with UN. For example, SLOWLY, and BULLY would be accepted and ABLE and UNACCEPTABLY would be rejected.

2. **[4]** Consider the following "proof" that for all odd positive integers n, $8 \mid n^2 - 1$.

$P(n): 8 \mid n^2 - 1$

Base case:

$n = 1$
$1^2 - 1 = 0$
$8 \mid 0$ since $0 = 8*0$, so our base case holds.

Inductive Step:

We need to prove that $\forall n \in N,\ P(n) \rightarrow P(n+1)$

Assume $P(n)$ is true, or that $8 \mid n^2 - 1$ (Inductive Hypothesis)
We must prove that $8 \mid (n+1)^2 - 1$

We know that n is odd, so $n = 2p-1$ for some integer p.

Since $8 \mid n^2 - 1$ there is some integer m such that $n^2 - 1 = 8m$.

Now let's consider $(n+1)^2 - 1$

$(n+1)^2 - 1$	$= n^2 + 2n + 1 - 1$	expand
	$= n^2 - 1 + 2n + 1$	re-order terms
	$= 8m + 2n + 1$	by the inductive hypothesis
	$= 8m + 2(2p-1) + 1$	since $n = 2p-1$
	$= 8m + 4p - 2 + 1$	expand
	$= 8m + 4p - 1$	add -2 and 1
	$= 8m + 8r$	$4p - 1 = 8r$, for some r
	$= 8(m + r)$	factor

Since $P(1)$ and $\forall n \in N,\ P(n) \rightarrow P(n+1)$, we can conclude that $\forall n \in N,\ P(n)$.

The above "proof" is invalid. What is wrong with the "proof"?

[8] 3. Prove that, for all **n ≥ 0**,

$$\sum_{i=0}^{n} i(i!) = (n+1)! - 1$$