CPSC 121,
2007 Summer
Quiz 3

Name: _____ Student ID: _____

Signature: _____

- You have **15 minutes** to write the **2 questions** on this examination.
- A total of 16 marks are available. The marks for each question are shown in square brackets to the left of the question number. **You may want to complete what you consider to be the easiest questions first!**
- Justify all of your answers.
- No notes or electronic equipment are allowed.
- Keep your answers short. If you run out of space for a question, you have written too much.
- Use the attached blank page for your rough work.
- Good luck!

Question	Marks
1	
2	
Total	

UNIVERSITY REGULATIONS:

- Each candidate should be prepared to produce, upon request, his/her university-issued ID.
- No candidate shall be permitted to enter the examination room after the expiration of one half hour or to leave during the first half hour of the examination.
- CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
    1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
    2. Speaking or communicating with other candidates.
    3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

[8] 1. Consider the sets:

$A = \{3, f, \{2\}\}$
$B = \{ x \mid (x \in \mathbf{Z}) \wedge (0 \leq x \leq |A|) \}$

and the function

$f: P(A) \rightarrow B$
$f(x) = |x|$

a) [2] What is P(A)? (Remember that P(A) is the power set of A)

P(A) = { {}, {3}, {f}, {{2}}, {3,f}, {3,{2}}, {f,{2}}, {3,f,{2}} }

b) [2] Is f injective? Why, or why not? (Remember that injective means the same as one-to-one)

No, f is not injective. f({3}) = f({{2}}) = 1

c) [2] Is f surjective? Why, or why not? (Remember that surjective means the same as onto)

Yes, f is surjective because every element in the co-domain has at least one pre-image.

B = {0,1,2,3}

f({}) = 0
f({3}) = 1
f({3,f}) = 2
f({3,f,{2}}) = 3

d) [2] Is f^{-1} a function? Why, or why not?

No, f^{-1} is not a function because $f^{-1}(1) = \{3\}$ and $f^{-1}(1) = \{f\}$ and in order for f^{-1} to be a function, it must map each element of its domain (B) to only one element in the co-domain (P(A)).

[8] 2. Use **an element proof** to show that, for any sets **A** and **B**:

$$(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$$

Hint: For any two sets **X** and **Y**, $X - Y = X \cap Y^c$

$$\begin{aligned}
(A \cup B) - (A \cap B) &= \{x \mid x \in (A \cup B) \cap (A \cap B)^c\} && \text{Def'n of set difference}\\
&= \{x \mid x \in (A \cup B) \wedge x \in (A \cap B)^c\} && \text{Def'n of intersection}\\
&= \{x \mid x \in (A \cup B) \wedge \sim(x \in (A \cap B))\} && \text{Def'n of set complement}\\
&= \{x \mid (x \in A \vee x \in B) \wedge \sim(x \in (A \cap B))\} && \text{Def'n of union}\\
&= \{x \mid (x \in A \vee x \in B) \wedge \sim(x \in A \wedge x \in B)\} && \text{Def'n of intersection}\\
&= \{x \mid (x \in A \vee x \in B) \wedge (x \notin A \vee x \notin B)\} && \text{De Morgan's}\\
&= \{x \mid ((x \in A \vee x \in B) \wedge x \notin A) \vee ((x \in A \vee x \in B) \wedge x \notin B)\} && \text{Dist. Law}\\
&= \{x \mid ((x \notin A \wedge x \in B) \vee (x \in A \wedge x \notin A)) \vee ((x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin B))\} && \text{Dist}\\
&= \{x \mid ((x \notin A \wedge x \in B) \vee F) \vee ((x \in A \wedge x \notin B) \vee F)\} && \text{Negation Law}\\
&= \{x \mid (x \notin A \wedge x \in B) \vee (x \in A \wedge x \notin B)\} && \text{Identity Law}\\
&= \{x \mid (x \in A \wedge x \notin B) \vee (x \notin A \wedge x \in B)\} && \text{Commutative Law}\\
&= \{x \mid (x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin A)\} && \text{Commutative Law}\\
&= \{x \mid (A - B) \vee (B - A)\} && \text{Def'n of set difference}\\
&= \{x \mid (A - B) \cup (B - A)\} && \text{Def'n of union}
\end{aligned}$$