CPSC 121,
2007 Summer
Quiz 2

Name: _____SOLUTIONS____ Student ID: _____

Signature: _____

- You have **20 minutes** to write the **3 questions** on this examination.
- A total of 20 marks are available. The marks for each question are shown in square brackets to the left of the question number. **You may want to complete what you consider to be the easiest questions first!**
- Justify all of your answers.
- No notes or electronic equipment are allowed.
- Keep your answers short. If you run out of space for a question, you have written too much.
- Use the back page for your rough work.
- Good luck!

Question	Marks
1	
2	
3	
Total	

UNIVERSITY REGULATIONS:

- Each candidate should be prepared to produce, upon request, his/her university-issued ID.
- No candidate shall be permitted to enter the examination room after the expiration of one half hour or to leave during the first half hour of the examination.
- CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
    1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
    2. Speaking or communicating with other candidates.
    3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

[6] 1. Consider the following definitions:

D: the set of all dogs

G(x): x is a golden retriever

W(x,y): x and y eat the same kind of dog food

F(x): x is female

a) Translate the following proposition into English

$\exists x \in D, \exists y \in D,\ G(x) \wedge G(y) \wedge W(x, y)$

At least two golden retrievers eat the same kind of dog food.

b) Translate the following proposition into predicate logic.

All golden retrievers eat the same kind of dog food.

$\forall x \in D,\ \forall y \in D,\ G(x) \wedge G(y) \rightarrow W(x,y)$

c) Negate the following proposition – write your answer in predicate logic.

$\exists x \in D,\ G(x) \wedge F(x)$

$\sim\exists x \in D,\ G(x) \wedge F(x)$
$= \forall x \in D,\ \sim[\ G(x) \wedge F(x)]$
$= \forall x \in D,\ \sim G(x) \vee \sim F(x)$

[5] 2. Prove or disprove: $\forall x \in R, \forall y \in R, x > y \rightarrow x^2 > y^2$

(Hint: Remember that R is the set of real numbers)

This is false, we'll give a counterexample to disprove it.
Let x = 1 and y = -3. x is greater than y, but x^2 is not greater than y^2.

[9] 3. Theorem: If **n** is even, then $\mathbf{n^3 + n^2}$ is even**.**

[2]. a) Translate the theorem into predicate logic.

$\forall n \in Z,\ 2|n \rightarrow 2|(n^3 + n^2)$

[7]. b) Prove the theorem.

WLOG, pick n to be an arbitrary integer.
We will assume that the antecedent is true and show that then the conclusion must be true.
So, we know that 2|n, so there exists some integer x such that 2x = n.

Therefore,

$n^3 + n^2 = (2x)^3 + (2x)^2$
$= 8x^3 + 4x^2$
$= 2(4x^3 + 2x^2)$

$4x^3 + 2x^2$ is some integer – let's call it y.

Then, $n^3 + n^2 = 2y$ where y is an integer, so therefore $2|(n^3 + n^2)$.
QED.