

CPSC 121,
2007 Summer
Practice Quiz 2

Name: _____ **SOLUTIONS** _____

Student ID: _____

Signature: _____

- You have **22 minutes** to write the **4 questions** on this examination.
- A total of 20 marks are available. The marks for each question are shown in square brackets to the left of the question number. **You may want to complete what you consider to be the easiest questions first!**
- Justify all of your answers.
- No notes or electronic equipment are allowed.
- Keep your answers short. If you run out of space for a question, you have written too much.
- The number in square brackets to the left of the question number indicates the number of marks allocated to that question.
- Use the attached blank page for your rough work.
- Good luck!

Question	Marks
1	
2	
3	
4	
Total	

UNIVERSITY REGULATIONS:

- Each candidate should be prepared to produce, upon request, his/her university-issued ID.
- No candidate shall be permitted to enter the examination room after the expiration of one half hour or to leave during the first half hour of the examination.
- **CAUTION:** candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
 1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
 2. Speaking or communicating with other candidates.
 3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

[4] 1. Consider the following definitions:

C: the set { Honda Civic, Mazda 3, Ford Focus, Jeep Cherokee, Chevrolet Cobalt }

L(x,y): the price of x is < the price of y

D(x,y): the price of x is \geq the price of y

The price of each car is:

- Honda Civic: \$25, 000
- Mazda 3: \$26, 000
- Ford Focus \$20, 000
- Jeep Cherokee \$40, 000
- Chevrolet Cobalt \$17, 000

For each of the following propositions

- Translate it into English
- State whether it is true or false, and show why or why not.

a) $\exists x \in C, \exists y \in C, L(x, y)$

There is a car that costs less than one of the other cars. This is true. Use a witness proof. Let $x =$ Ford Focus and $y =$ Mazda 3. $L(x,y)$ is true, therefore the proposition is true.

b) $\forall x \in C, \exists y \in C, D(x, y)$

For every car, there exists a car that is more expensive or equally expensive. This is true. Let x be an arbitrary car in the set C , and let $y=x$. $D(x,y)$ is true – therefore the proposition is true.

[6] 2. Translate the following propositions into predicate logic.

Assume that:

T: is the set of girls on Meghan's soccer team

G: is the set of games Meghan's soccer team has played this year

S(x,y): player x scored a goal in game y

W(y): Meghan's soccer team won game y.

a) Every player on Meghan's soccer team has scored at least one goal this year.

$$\forall p \in T, \exists g \in G, S(p, g)$$

b) Meghan's soccer team wins only if Katie scores.

$$\forall g \in G, [W(g) \rightarrow \exists t \in T, t = \text{Katie} \wedge S(t, g)]$$

[3] 2. Prove $\sim(\forall x \in \mathbf{R}, x^2 > x)$ (Hint: Remember that \mathbf{R} is the set of real numbers)

$$\sim(\forall x \in \mathbf{R}, x^2 > x) = \exists x \in \mathbf{R}, x^2 \leq x$$

Prove $\exists x \in \mathbf{R}, x^2 \leq x$ by witness.

Let $x = .5$, then $x^2 = .25$ and $x^2 \leq x$.

QED.

[7] 2. Prove: If m and n are rational, then m^2n^2 is rational. You will get partial marks for writing the correct structure of the proof (even if you do not fill in the details). (Hint: Remember that a rational number can be expressed as a/b , where a and b are integers)

$RA(x)$: x is rational

$$\forall m, n \in \mathbf{R}, RA(m) \wedge RA(n) \rightarrow RA(m^2n^2)$$

Pick an arbitrary m and n

Assume $RA(m) \wedge RA(n)$ and then prove $RA(m^2n^2)$.

Since $RA(m)$, we can write m as a/b where a and b are integers and $b \neq 0$.

Since $RA(n)$, we can write n as c/d where c and d are integers and $d \neq 0$.

$$\text{Then } m^2n^2 = \left(\frac{a}{b}\right)^2 \left(\frac{c}{d}\right)^2 = \frac{a^2c^2}{b^2d^2}$$

Since, a , b , c and d are integers, a^2 , b^2 , c^2 and d^2 are integers. Since a^2 and c^2 are integers, a^2c^2 is an integer. Since b^2 and d^2 are integers, b^2d^2 is an integer. Since $b \neq 0$ and $d \neq 0$, $b^2 \neq 0$ and $d^2 \neq 0$, and therefore $b^2d^2 \neq 0$. Since we know that a^2c^2 is an integer, and b^2d^2 is an integer that is not

equal to 0, $\frac{a^2c^2}{b^2d^2}$ is rational, and therefore m^2n^2 is rational.

QED.