Practice problems

1. Does the order of quantifiers matter?
   - Are these equivalent?
     - $\forall x \in N, \exists y \in N, y = x + 1$
     - $\exists y \in N, \forall x \in N, y = x + 1$

No, they are not. The first proposition is true and the second one is false.

2. Express "Every first year student at UBC is passing some CS course" with predicates and quantifiers

a) when the domain for students is first year UBC students and the domain for courses is CS courses

Let U be the set of first year UBC students, C be the set of CS course and P(x,c) mean student x is passing course c

$\forall s \in U, \exists c \in C, P(s,c)$

a) when the domain for students is all UBC students and the domain for courses is all UBC courses.

Let T be the set of all UBC students, D be the set of all UBC courses, P(x,c) mean student x is passing course c, F(x) mean student x is in first year and CS(x) mean course x is a CS course

$\forall s \in T, F(s) \rightarrow (\exists c \in D, CS(c) \wedge P(s,c))$

Practice problems

E(x): x is even

G(x): x > 10

Prove or disprove

- $\forall x \in N, E(x) \rightarrow G(x)$

Disproof, let x = 6. E(x) is true but G(x) is false, therefore the proposition $\forall x \in N, E(x) \rightarrow G(x)$ is false.

- $\exists x \in N, E(x) \rightarrow G(x)$

Witness proof, let x = 13. E(x) is false, therefore the proposition $\exists x \in N, E(x) \rightarrow G(x)$ is true.