Do problems 0 and 1 and any two of 2, 3, or 4. Graded on a scale of 100 points.

0. (**5 points**) Your name: ______________________ Your student #: ______________

1. (**35 points**) (Sipser exercise 1.47)
Let $\Sigma = \{\texttt{1}, \texttt{\#}\}$ and let

$$A \quad = \quad \{w \mid w = x_1\texttt{\#}x_2\texttt{\#}\cdots\texttt{\#}x_k,\ k \geq 0, \text{each } x_i \in \texttt{1}^* \text{ and } (i \neq j) \Rightarrow (x_i \neq x_j)\}$$

In English, A is the set of all strings consisting of zero or more strings of `1`'s separated by `#`'s such that no two of these strings of `1`'s have the same length. For example `1`, `1#11#111`, `1111##11#1111111` and `111#1111#11111#111111` are in A, but `1#1` and `1#11#111#11` are not.

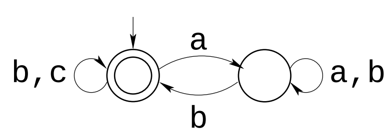
Prove that A is not regular.

2. (**30 points**)

(a) (**10 points**) Give a DFA that recognizes the language $\mathtt{a}(\mathtt{a} \cup \mathtt{b})^*\mathtt{b} \;\cup\; \mathtt{b}(\mathtt{b} \cup \mathtt{a})^*\mathtt{a}$.
The input alphabet is $\{\mathtt{a}, \mathtt{b}\}$. Drawing a state diagram for your DFA is sufficient.

(b) (**10 points**) Give a NFA that recognizes the language $(\mathtt{ab}^*)^*\mathtt{c} \;\cup\; (\mathtt{ab})^*$.
The input alphabet is $\{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$. Drawing a state diagram for your NFA is sufficient.

(c) (**10 points**) Give a regular expression corresponding to the NFA:



Answer: ______________________________

3. (**35 points**) Let B be any language. Define

$$f(B) \quad = \quad \{w \mid \exists x \in B.\, x = ww^{\mathcal{R}}\}$$

where $x^{\mathcal{R}}$ denotes the reverse of string x. For example,

$$f(\{\texttt{cattac}, \texttt{doggod}, \texttt{mouseesoum}\}) \quad = \quad \{\texttt{cat}, \texttt{dog}, \texttt{mouse}\}$$

Show that if B is any regular language, then $f(B)$ is regular as well. It is sufficient to describe the construction of a DFA, NFA or regular expression for $f(B)$ and/or use closure properties that we have already proven. You don't need to give a formal proof that your construction is correct.

4. (**35 points**) Ever had a broken keyboard that dropped or repeated characters? If so, this problem is for you. Let Σ be a finite alphabet, and let $RE(\Sigma)$ denote all regular expressions over strings in Σ^*. Define $\mathit{flakeyKeys} : \Sigma^* \rightarrow RE(\Sigma*)$ as shown below

$$\begin{aligned} \mathit{flakeyKeys}(\epsilon) &= \epsilon \\ \mathit{flakeyKeys}(x \cdot c) &= x \circ c^*, \quad \text{for any } c \in \Sigma \end{aligned}$$

In other words, $\mathit{flakeyKeys}(x)$ maps the string x to a regular expression that matches any string that can be derived from x by dropping or repeating symbols. For example, $\mathit{flakeyKeys}(\texttt{cat})$ is the regular expression $\texttt{c}^*\texttt{a}^*\texttt{t}^*$

Let C be any language. Define

$$\mathit{flakeyKeys}(C) \quad = \quad \{w \mid \exists x \in C.\ w \in \mathit{flakeyKeys}(x)\}$$

Show that if C is regular, then $\mathit{flakeyKeys}(C)$ is regular as well. It is sufficient to describe the construction of a DFA, NFA or regular expression for $\mathit{flakeyKeys}(C)$ and/or use closure properties that we have already proven. You don't need to give a formal proof that your construction is correct.