

Midterm 2 Review

March 19th, 2012

1 Topics

- i. Predicate Logic
- ii. Formal Proofs & Techniques
- iii. Deterministic Finite State Automata
- iv. Flipflops, Circuit Memory
- v. Induction

2 Examples

I Use the following to answer each question in two equivalent different ways, one existential, one universal:

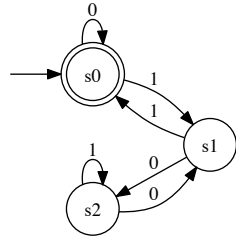
- $\text{Dog}(x) : x$ is a dog
- $\text{Green}(x) : x$ is green-coloured
- $\text{LargerThan}(x, y) : x$ is larger than y
- $\text{SameSize}(x, y) : x$ is the same size as y
- A , the set of all animals

- (a) There is an animal that is green.
- (b) No dogs are green.
- (c) There is a non-dog that is larger than a dog.

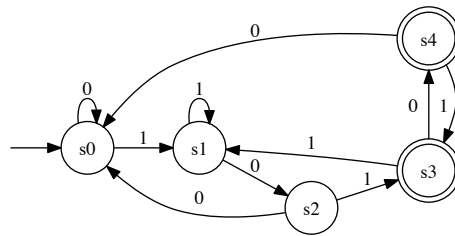
II Convert these proofs to predicate statements and outline how you would approach the proof:

- (a) Is it true that if a and b are real numbers, $a \neq 0$, then there is a unique real number r s.t. $ar + b = 0$
- (b) Prove that if m is a perfect square, $m + 2$ is not a perfect square
- (c) Prove that if x^3 is irrational, x is irrational.

III Describe the languages the following DFA's accept:



(a)



(b)

IV Consider the following Circuit questions:

- (a) How does one get an 'initial' value for a flip-flop? How do we prevent using this value each clock-tick?
- (b) Design a circuit that counts even numbers up to 6 (i.e. 2, 4, 6, 0)

V Induction Questions:

- (a) Prove $P(n) \leftrightarrow 1 + 2 + 4 + \dots + 2^n = 2^{n+1} - 1$
- (b) $x, y, n \in \mathbb{N}$
If $x < y$, then $x^n < y^n$