CpSc 421 Midterm **45 minutes**

Bug bounties are in effect. If you find an error that affects the solution of the exam, raise your hand and bring it to my attention. Please note that I don't understand whispers so you might want to write a one or two sentence description of the problem as I head to your seat. I will write corrections on the white board. Bug bounties will be awarded in midterm points.

If you find a minor spelling or grammatical error, please do not disturb everyone else by reporting it. Such errors can be reported to the newsgroup after the exam. I will give double points to the first person who reports an error. If two reports of the same error are posted close enough together that they were plausibly written at the same time, I will give points to both.
Good luck!

1. **(40 points):** Let $\#\mathsf{a}(w)$ be the number of a's in w and $\#\mathsf{b}(w)$ be the number of b's in w. For each language below, determine whether or not it is regular. Give a brief justification for each answer.

   (a) **(20 points):** $A_1 = \{w \in \{a,b\}^* |\ \#\mathsf{a}(w) - \#\mathsf{b}(w) \text{ is divisible by } 3\}$

   (b) **(20 points):** $A_2 = \{w \in \{a,b\}^* |\ \#\mathsf{a}(w) - \#\mathsf{b}(w) < 3\}$

   (c) **(10 points, Extra Credit):** $A_3 = \left\{w \in \{a,b\}^* \;\middle|\; |\#\mathsf{a}(w) - \#\mathsf{b}(w)| > 3\right\}$

2. **(20 points):** Let $B = \{w \in \{a,b\}^* |\ \#\mathsf{a}(w) = 2\#\mathsf{b}(w)\}$. Is B context-free? Give a short justification for your answer.

3. **(20 points):** Let B be the balanced parentheses language:

   $$S \quad \rightarrow \quad \epsilon \mid \mathbf{[}\, S \,\mathbf{]} \mid SS$$

   Let $U = \sim B$ be the language of strings where parentheses are *not* properly balanced. Is U context-free? Give a 4-5 sentence justification for your answer.

4. **(20 points):** Let G be a CFG in Chomsky normal form. Let w be a string in $L(G)$, and let n be the number of steps in a derivation of w. Prove that:

   $$n \quad = \quad 2|w| - 1$$

   Hint: I promised that there would be no long proofs required on this test.