



Figure 1: NFA for question 2

For each question, I've given some number of "points." The actual midterm will have 100 points worth of questions.

1. **(100 points):** For each language below, state whether or not it is regular, and whether or not it is context free. Give a short (1-3 sentences or formulas) justification for each answer.
 - (a) **(25 points):** $\{w \in \{a, b\}^* \mid \exists n \in \mathbb{Z}. w = a^n b^{2n}\}$
 - (b) **(25 points):** $\{w \in \{a, b\}^* \mid \exists n \in \mathbb{Z}. w = (abb)^n\}$
 - (c) **(25 points):** $\{w \in \{a, b\}^* \mid \#a(w) = 2\#b(w)\}$
where $\#a(w)$ is the number of "a" symbols in w and likewise for $\#b(w)$.
 - (d) **(25 points):** $\{w \in \{a, b\}^* \mid \exists n \in \mathbb{Z}. w = a^n b^{n^2}\}$

2. **(30 points):** For each string below, determine whether or not it is accepted by the NFA shown in figure 1.
 - (a) **(5 points):** a
 - (b) **(5 points):** aa
 - (c) **(5 points):** b
 - (d) **(5 points):** bb
 - (e) **(5 points):** abab
 - (f) **(5 points):** baba

3. **(20 points):** Write a regular expression or Kozen pattern that recognizes all strings over the alphabet $\{a, b, c\}$ where every b is preceded immediately by an a , and every c is either preceded two symbols earlier by a a or is followed immediately by another c . For example the strings: a, aa, ab, abc, ϵ , and acc are in the language, but b , and acb are not.

4. **(12 points):** True or false

- (a) Regular expressions are closed under union.
- (b) Regular expressions are closed under complement.
- (c) Context free languages are closed under asteration.
- (d) Context free languages are closed under intersection.
- (e) Regular languages are a subset of the deterministic context free languages.
- (f) Every language is context free.

5. **(question schemas):** Here are some forms questions could take:

- (a) Given the CFG below, determine which of the following two strings it generates and which it does not. Give the derivation for the string that it accepts, and explain why the other string cannot be derived.
- (b) Given the NPDA in figure X, determine which of the following two strings it accepts and which it rejects. List the sequence of configurations that the machine goes through for the string that it accepts.