CPSC 121 Quiz 4
Wednesday, 2012 July 25

Name: ____________________ Student ID: ____________________
Signature: ____________________
Your signature acknowledges your understanding of and agreement to the rules below.

– You have 40 minutes to write the 5 questions on this examination. A total of 20 marks are available.

– You may have as an aide up to 3 textbooks and a 3 inch stack of paper notes and nothing else. **No electronic devices allowed**; so, no cell phones and no calculators.

– Keep your answers short. If you run out of space for a question, you have likely written too much.

– The number in square brackets to the left of the question number indicates the number of marks allocated for that question. Use these to help you plan your use of time on the exam.

– Clearly indicate your answer to each problem. If your answer is not in the provided blank, then indicate where the answer is, and at the answer's location indicate the question it addresses.

– **Good luck!**

Question	Marks
1	
2	
3	
4	
5	
Total	

[2] 1. In proving the following theorem with direct proof techniques, you would choose values for a and b. Which of p, q, and r can your choices for a's and b's values depend on? (For example, if all of these were integers and b could be chosen to be $p * q + r$, then it can depend on all of p, q, and r.)

Theorem:

$$\exists a \in A, \forall p \in B, \forall q \in A, R(a,p,q) \rightarrow \exists b \in B, N(a,b) \wedge \forall r \in B, P(q,b,a,r)$$

a's value can depend on: (circle **ALL** correct answers) p q r NONE OF THESE

b's value can depend on: (circle **ALL** correct answers) p q r NONE OF THESE

[2] 2. Imagine you were instead going to prove the following theorem by contradiction (as your first step). Write the appropriate assumption with any negations moved "inward" as much as possible. When you're done, negation(s) should appear only on predicates—e.g., $\sim M(a,b)$ or $b \neq c$—and not on quantifiers or parenthesized expressions.

You need not show each step of moving negations inward (but you can, if you wish!).

Theorem:

$$\exists a \in A, \forall p \in B, \forall q \in A, R(a,p,q) \rightarrow \exists b \in B, N(a,b) \wedge \forall r \in B, P(q,b,a,r)$$

[4] 3. For each of the following theorems, indicate the most complete "direct" proof approach—no proof by contradiction, no use of logical equivalences—that you can by writing the letters of the techniques listed below in the order you would use them. Ignore any blanks you don't need.

List of proof techniques:

A. Witness proof (in Epp: constructive proof of existence)
B. "Without loss of generality" proof (in Epp: generalizing from the generic particular)
C. Antecedent assumption.
D. Equality or inequality proof.

So, for a proof that starts as a witness proof, then uses "without loss of generality" twice, and then antecedent assumption: write **A** in blank #1, **B** in blank #2, **B** in blank #3, **C** in blank #4, and nothing in blank #5.

Theorem: $\exists x \in S, (x \geq 5) \rightarrow \forall y \in S, P(x, y)$.

Strategy steps: #1 _____ #2 _____ #3 _____ #4 _____ #5 _____

Theorem: $\forall v_1 \in \mathbb{R}^+, \forall v_2 \in \mathbb{R}^-, \exists c \in \mathbb{R}, v_1 * v_2 \leq c$

Strategy steps: #1 _____ #2 _____ #3 _____ #4 _____ #5 _____

[3] 4. While cleaning a robot purchased for your parents' farm, you accidentally trigger a damaged recording of a human-like female alien proving some theorem.

Below is the remaining text of the proof. Write out the theorem **in as much detail as possible**. (Use the predicates *Old* and *Green* if they help.)

> Without loss of generality, let y be a member of J. We proceed in two cases. Case 1: Assume y is old. Then, let $p =$ QG (also a member of J). We now show that $mc(p) < mc(y)$. [*part of the proof is impossible to hear*] Case 2: Assume y is green. Then, let $p =$ LS (also a member of J). We now show that $mc(p) < mc(y)$. [*and the recording ends*]

[9] 5. Consider the following theorem: for every integer $b \geq 2$, there are integers k and n with $0 < k < n$ such that $\frac{k}{n}$ is representable in base b with a finite number of digits.

[2] a. Translate the theorem to predicate logic. (Use $R(a, b)$ to mean "the fraction a is representable in base b with a finite number of digits.")

Theorem:

[7] b. Prove the theorem. Use a direct proof, making the structure of your proof **clear**. (Partial credit is available for getting the structure correct.)

BONUS: Earn up to 2 bonus points by doing one or more of these problems.

- Translate and prove this theorem: "For every integer $b \geq 2$ and integer k, there is an integer n with $0 < k < n$ such that $\frac{k}{n}$ is representable in base b with a finite number of digits."
- Translate and prove this other similar theorem: "For every integer $b \geq 2$, there are integers k and n with $0 < k < n$ such that $\frac{k}{n}$ is *not* representable in base b with a finite number of digits."

BONUS? Maybe worth a small amount of bonus points: In the robot message problem, tell us who the robot and the human alien are and what y, J, p, QG, the $mc(\cdot)$ function, and LS are.