

CPSC 121: Models of Computation
Quiz #2: Section BCS, 2009 March 19

Name: _____ Student ID: _____

Signature: _____ Tutorial Section: _____

- You have **30 minutes** to write the 3 questions on this quiz.
- A total of **16 marks** are available. You may want to complete what you consider to be the easiest questions first!
- Ensure that you clearly indicate a single legible answer for each question.
- You are allowed a single 8.5" x 11" reference sheet. The sheet must have your name on it and may contain any content you like. Otherwise, no notes, aides, or electronic equipment are allowed.
- Good luck!

UNIVERSITY REGULATIONS

1. Each candidate must be prepared to produce, upon request, a UBCCard for identification.
2. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
3. No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
4. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:
 - having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;
 - speaking or communicating with other candidates; and
 - purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
5. Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
6. Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1 Creating Induction Proofs [6 marks]

Part of the marks for this question will be given for writing the framework of the proof clearly and completely. In other words, you will be marked on the form of your proof as well as on its content.

Prove by induction on n that for every positive integer n , $n^3 + 2n$ is divisible by 3. Recall that we say n is divisible by d (equivalently, d divides n) if and only if there is an integer k such that $d \cdot k = n$.

2 Critiquing Induction Proofs [5 marks]

Consider the following induction proof. The proof contains one or more flaws; however, the theorem to be proved is true and provable by induction. Your job is to create a correct proof. The easiest way to do so is likely to be by crossing out and correcting erroneous lines.

Note: the definitions and statement of the theorem are *not* flawed.

Definitions: Consider the following sequence: $p_0 = 1$, $p_1 = 1$, and for all integers $n \geq 2$, $p_n = p_{\lceil n/3 \rceil} + p_{\lfloor 2n/3 \rfloor} + 1$.

Also note that for all integers n , $\lceil n/3 \rceil + \lfloor 2n/3 \rfloor = n$.

Theorem: For all positive integers n , $p_n = 2n - 1$.

Proof: We proceed by induction.

Base Case:

We use $n = 1$ as a base case:

$$\begin{aligned} p_1 &= p_{\lceil 1/3 \rceil} + p_{\lfloor 2 \cdot 1/3 \rfloor} + 1 \\ &= p_1 + p_0 + 1 \\ &= 1 + 1 + 1 \\ &= 3 \checkmark \end{aligned}$$

Induction Hypothesis: Assume for an arbitrary integer $1 \leq k < n$ that $p_k \leq 2k - 1$, for integers $n \geq 2$.

Inductive Step: Given the Induction Hypothesis, we must show that $p_n \leq 2n - 1$.

Since $n \geq 2$, we know by our induction hypothesis that $p_n = p_{\lceil n/3 \rceil} + p_{\lfloor 2n/3 \rfloor} + 1$.

Now, we establish that the induction hypothesis applies to $p_{\lceil n/3 \rceil}$ and $p_{\lfloor 2n/3 \rfloor}$.

Since $n \geq 2$, $n/3 \geq 2/3 > 0$. Therefore, $\lceil n/3 \rceil \geq 1$.

Similarly, since $n \geq 2$, $2n/3 \geq 4/3 \geq 1$. Therefore, $\lfloor 2n/3 \rfloor \geq 1$.

Since $\lceil n/3 \rceil + \lfloor 2n/3 \rfloor = n$ and both parts are at least 1, both are also less than n (or the total would be greater than n).

Thus, the induction hypothesis applies to both cases.

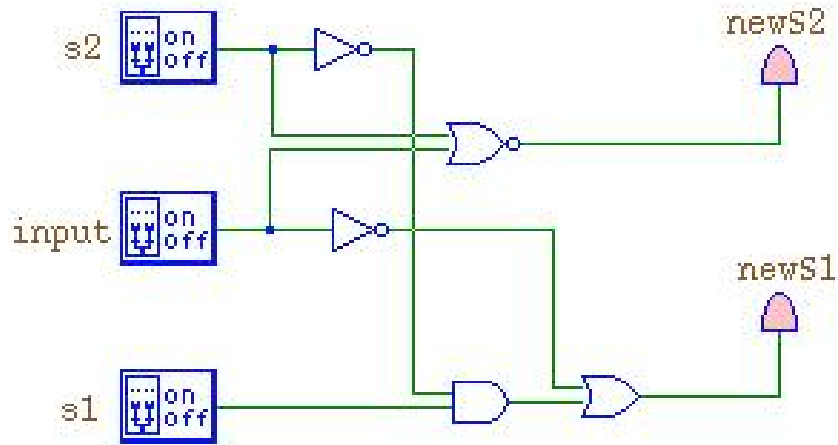
Therefore:

$$\begin{aligned} p_n &= p_{\lceil n/3 \rceil} + p_{\lfloor 2n/3 \rfloor} + 1 \\ &\leq 2\lceil n/3 \rceil - 1 + 2\lfloor 2n/3 \rfloor - 1 + 1 \\ &= 2(\lceil n/3 \rceil + \lfloor 2n/3 \rfloor) - 1 \\ &= 2n - 1 \end{aligned}$$

This completes our inductive step. Therefore, for all non-negative integers n , $p_n \leq 2n - 1$. □

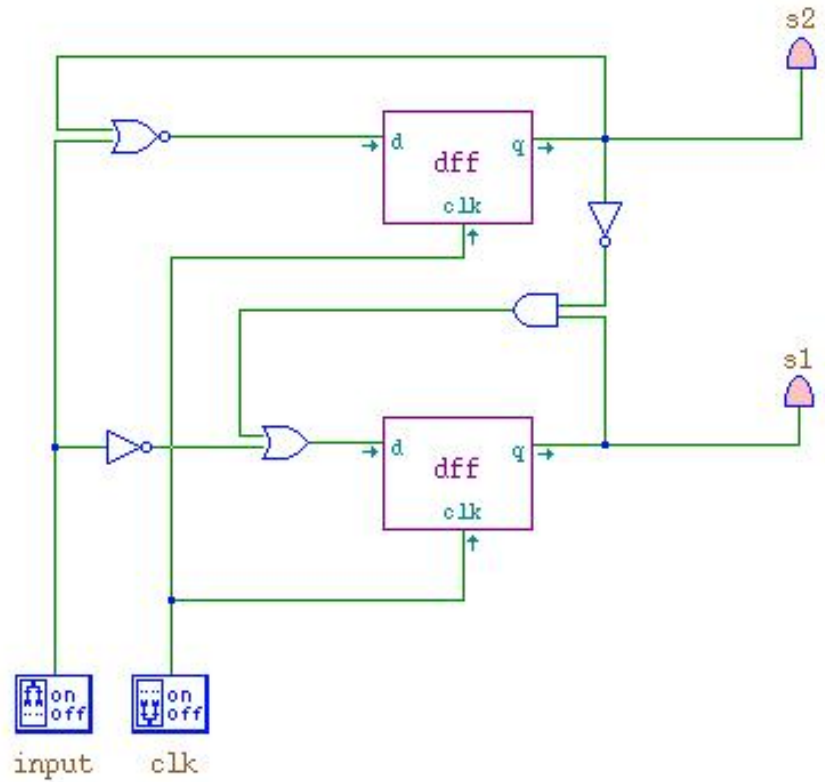
3 Sequential Circuits [5 marks]

1. Fill in the following truth table for the combinational circuit shown. (Note that this is the key combinational component of the sequential circuit below.)



<i>input</i>	<i>s1</i>	<i>s2</i>	<i>newS1</i>	<i>newS2</i>
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

2. The table below shows the sequence of input values to the following sequential circuit each time the clock ticks (goes from low to high). Fill in the appropriate value for *s1* and *s2* immediately **after** the clock ticks for each row. Use 1 for high/true, 0 for low/false, and *U* for undefined (yellow in TkGate).



Note that one row is given (to break the problem into two parts).

<i>input</i>	<i>s1</i>	<i>s2</i>
0		
1		
0	1	1
0		
0		
1		
1		
0		
0		
1		