CPSC 121: Models of Computation
Quiz #1: Section 203, February 4, 2009

Name: _____ Student ID: _____

Signature: _____

- You have **30 minutes** to write the 4 questions on this quiz.
- A total of **16 marks** are available. You may want to complete what you consider to be the easiest questions first!
- Ensure that you clearly indicate a single legible answer for each question.
- You are allowed a single 8.5" x 11" reference sheet. The sheet must have your name on it and may contain any content you like. Otherwise, no notes, aides, or electronic equipment are allowed.
- Good luck!

UNIVERSITY REGULATIONS

1. Each candidate must be prepared to produce, upon request, a UBCcard for identification.

2. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

3. No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

4. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:

- having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;
- speaking or communicating with other candidates; and
- purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.

5. Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

6. Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1 Describing Relationships with Predicate Logic [4 marks]

Definitions: Let I be the set of all valid input text. Let P be the set of all valid Java programs. Let Runs(p, i) mean that program p runs to completion on input text i. (In other words, we start up program p and then type in the input text i. If the program ever stops, it runs to completion. If it never stops for some reason, then it does not run to completion.)

Using these definitions, translate the following statement into predicate logic:

Not every program runs to completion on every input.

2 Critiquing Propositional Logic Proofs [4 marks]

Consider the following propositional logic proof. Some steps in the proof are invalid. Circle the step or steps that are invalid and explain why they are invalid.

(Note: any explanation that clearly describes a flaw in the step is acceptable, but the easiest explanations will typically indicate how a logical equivalence or rule of inference was applied inappropriately.)

1.	$b \vee \overline{c}$	premise
2.	$(c \wedge d) \vee e$	premise
3.	$\overline{e} \wedge (h \rightarrow g)$	premise
4.	$\overline{a \wedge b}$	premise
5.	$\overline{a} \vee \overline{b}$	by De Morgan on 4.
6.	$a \rightarrow \overline{b}$	by definition of condition on 5.
7.	$\overline{e}$	by specialization of 3.
8.	$c \wedge d$	by disjunctive syllogism on 7. and 2.
9.	$(c \wedge e) \vee (d \wedge e)$	by distribution on 2.
10.	c	by specialization of 8.
11.	b	by disjunctive syllogism on 10. and 1.
12.	$\overline{a}$	by modus tollens on 11. and 6.

3 Number Representation [4 marks]

Suppose we are willing to allocate 64 bits to represent integers. Consider the trade-offs (i.e., the pros and cons) of choosing between signed binary and an encoded representation, such as 16 digits of binary coded decimal (BCD).

For each of signed binary and BCD, suggest one advantage and one disadvantage it has compared to the other. HINT: Factors to consider include the range of numbers that can be represented and the difficulty in designing circuits to do basic operations such as input/output and simple arithmetic (addition/subtraction).

4 Circuit Design [4 marks]

Design a circuit using only inverters and two-input AND, OR, and XOR gates that takes as input three signals a_3, a_2, and a_1 and produces two outputs x_1 and x_0. If all of the inputs are 0, then all of the outputs should be 0. Otherwise, the output, x_1x_0, should be the unsigned, binary encoding of the smallest i for which a_i is 1. For example, if $a_3 = 1$, $a_2 = 1$, and $a_1 = 0$ then $x_1 = 1$ and $x_0 = 0$. HINT: This can be done using 6 gates, two of which are inverters.