

CPSC 121: Models of Computation  
Quiz #1: Section **202**, 2009 February 4/5

Name: \_\_\_\_\_ Student ID: \_\_\_\_\_

Signature: \_\_\_\_\_

- You have **30 minutes** to write the 4 questions on this quiz.
- A total of **16 marks** are available. You may want to complete what you consider to be the easiest questions first!
- Ensure that you clearly indicate a single legible answer for each question.
- You are allowed a single 8.5" x 11" reference sheet. The sheet must have your name on it and may contain any content you like. Otherwise, no notes, aides, or electronic equipment are allowed.
- Good luck!

## UNIVERSITY REGULATIONS

1. Each candidate must be prepared to produce, upon request, a UBCcard for identification.
2. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
3. No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
4. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:
  - having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;
  - speaking or communicating with other candidates; and
  - purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
5. Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
6. Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

## 1 Describing Relationships with Predicate Logic [4 marks]

Definitions: Let  $I$  be the set of all valid input text. Let  $P$  be the set of all valid Java programs. Let  $\text{Runs}(p, i)$  mean that program  $p$  runs to completion on input text  $i$ . (In other words, we start up program  $p$  and then type in the input text  $i$ . If the program ever stops, it runs to completion. If it never stops for some reason, then it does not run to completion.)

Using these definitions, translate the following statement into predicate logic:

**There are programs that run to completion on some inputs but not on others.**

## 2 Critiquing Propositional Logic Proofs [4 marks]

Consider the following propositional logic proof. Some steps in the proof are invalid. Circle the step or steps that are invalid and explain why they are invalid.

(Note: any explanation that clearly describes a flaw in the step is acceptable, but the easiest explanations will typically indicate how a logical equivalence or rule of inference was applied inappropriately.)

- |     |                                       |   |
|-----|---------------------------------------|---|
| 1.  | $\sim(a \vee d)$                      | premise   |
| 2.  | $e \rightarrow r$                     | premise   |
| 3.  | $((d \vee b) \wedge b) \rightarrow a$ | premise   |
| 4.  | $e \vee d$                            | premise   |
| 5.  | $\sim a \wedge \sim d$                | De Morgan's on 1                                |
| 6.  | $\sim a$                              | specialization on 5                             |
| 7.  | $\sim((d \vee b) \wedge b)$           | modus tollens on 3 and 6                        |
| 8.  | $\sim(d \vee b) \vee \sim b$          | De Morgan's on 7                                |
| 9.  | $(\sim d \wedge \sim b) \vee \sim b$  | De Morgan's on 8                                |
| 10. | $\sim d \wedge \sim b$                | absorption on 9                                 |
| 11. | $\sim d$                              | specialization on 10                            |
| 12. | $e$                                   | elimination (disjunctive syllogism) on 11 and 4 |

### 3 Number Representation [4 marks]

You are designing a 64 bit unique ID code for each person in the world. A code is composed of two numbers: a unique number for each country (the person's country of primary residence) and a unique number for each person. When a code is displayed in hexadecimal, a human reader should be able to easily read the code and, in particular, quickly identify just the country part.

Decide how many of the 64 bits will be used for the "country number" and how many for the "person number". Justify why your answer: (1) includes enough bits for the country, (2) includes enough bits for the person, and (3) makes the code readable.

You may find this list of approximate powers of two helpful:

- $2^0 = 1$
- $2^{10} \approx 1000$
- $2^{20} \approx 1,000,000$
- $2^{30} \approx 1,000,000,000$
- $2^{40} \approx 1,000,000,000,000$
- $2^{50} \approx 1,000,000,000,000,000$
- $2^{60} \approx 1,000,000,000,000,000,000$

#### **4 Circuit Design [4 marks]**

Build a “unanimous decision” circuit with three inputs  $x$ ,  $y$ , and  $z$ . The circuit has two outputs  $a$  and  $v$ . Output  $a$  is true if and only if the three inputs agree (are all true or all false). Output  $v$  matches the value of the inputs whenever  $a$  is true but does not matter (may be anything you like) when  $a$  is false.