

CPSC 121,
2005/6 Winter Term 2, Section 203
Quiz 2

Name: SAMPLE_____

Student ID: _____

Signature: _____

- You have **30 minutes** to write the **3 questions** on this examination.
- A total of 40 marks are available. **You may want to complete what you consider to be the easiest questions first!**
- Justify all of your answers.
- No notes or electronic equipment are allowed.
- Keep your answers short. If you run out of space for a question, you have written too much.
- The number in square brackets to the left of the question number indicates the number of marks allocated to that question.
- Use the attached blank page for your rough work.
- Good luck!

Question	Marks
1	
2	
3	
Total	

UNIVERSITY REGULATIONS:

- Each candidate should be prepared to produce, upon request, his/her university-issued ID.
- No candidate shall be permitted to enter the examination room after the expiration of one half hour or to leave during the first half hour of the examination.
- CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
 1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
 2. Speaking or communicating with other candidates.
 3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

[15] 1. Jubjubs have a height and a size. Jubjubs are formed in three ways:

(1) A new Jubjub of height 1 and size 1 appears every afternoon.

(2) Two Jubjubs of different heights can form a new Jubjub. The new Jubjub's size is the sum of the two parents' sizes and its height is the height of the taller parent.

NOTE: one of the two parents must be the same height as the child!

(3) Two Jubjubs of the same height can form a new, taller Jubjub. The new Jubjub's size is the sum of the two parents' sizes and its height is one larger than its parents' height.

NOTE: one of the two parents must be \leq half the size of the child!

For example:

- Using (2), a Jubjub of height 5 and size 41 and a Jubjub of height 3 and size 9 can form a Jubjub of height 5 and size 50.

- Using (3), a Jubjub of height 3 and size 20 and a Jubjub of height 3 and size 8 can form a Jubjub of height 4 and size 28.

Using induction on the size of a Jubjub, prove the following for every Jubjub, where h is the Jubjub's height and s is the Jubjub's size:

$$h \leq (\log_2 s) + 1$$

Help with logs: $\log_2 1 = 0$. $(\log_2 n) + 1 = \log_2 (2n)$.

Hint: proceed in two cases for your inductive step based on how the Jubjub was formed!

[10] 2. Prove the following for arbitrary sets A , B , and C : $(A \cap B \subseteq C) \rightarrow B \subseteq \overline{A} \cup C$. Your proof may be in a combination of English and logic but make sure that it's clear what the steps of your proof are and how each step follows from the previous steps.

[15] 3. Consider the set $A = \{\text{hello}, 3, \emptyset\}$, its power set $P(A)$, and its Cartesian product with itself $A \times A$. For each of the following functions, give an example value from the domain and its corresponding image. Then, say whether the function is (**Y**) or is not (**N**) injective, surjective, or bijective. The first row is filled in as an example.

The function f	Pre-image	Image	Injective? (Y or N)	Surjective? (Y or N)	Bijective? (Y or N)
$f : A \rightarrow A$ maps values onto themselves: $f(x) = x$.	hello	hello	Y	Y	Y
$f : A \rightarrow (A \cup P(A))$ maps values onto themselves: $f(x) = x$.					
$f : A \times A \rightarrow A$ picks out the first element from a 2-tuple					
$f : A \rightarrow P(A)$ maps elements into sets: $f(x) = \{x\}$.					
$f : P(A) \rightarrow P(A)$ “flips” sets: $f(B) = A - B$					
f takes 2-tuples of elements from A and maps them into subsets of A . (You pick the particular function.)					