The actual exam will be open book, open notes, open homework and solutions, closed mouth. No calculators, laptop computers, PDA's etc.

1. **(30 points):** Each of the three languages described below is regular. Show this. For one language, you should construct a DFA; for another, you should construct an NFA; and for the remaining language, you should write a regular expression. You can choose which method you use with which language.

   (a) $\{w \in \{\mathtt{0},\mathtt{1}\}^* \mid w$ is the binary representation of a number that is divisible by four.$\}$.

   (b) $\{w \in \{\mathtt{0},\mathtt{1}\}^* \mid w$ is the binary representation of a number that is divisible by five.$\}$.

   (c) $\{w \in \{\mathtt{0},\mathtt{1}\}^* \mid \exists x, y, z.\ (w = xyz) \wedge (\#\mathtt{1}(y) = (\#\mathtt{0}(y) + 3))\}$,
   where $\#\mathtt{0}(y)$ is the number of `0`'s in y, and $\#\mathtt{1}(y)$ is the number of `1`'s.
   In English, this says that w contains a substring (i.e. y) that has three more `1`'s than `0`'s.

   For parts (a) and (b), assume that the string is entered most significant bit first, i.e. `0101` represents the decimal value 5.

2. **(30 points):** Let A and B be regular languages over some alphabet Σ. Let C be the language:

$$C \quad = \quad \{w \mid \exists x, y, z \in \Sigma^*.\ (w = xz) \wedge (xyz \in A) \wedge (y \in B)\}$$

   Show that C is regular.
   An acceptable answer will construct a DFA, NFA, or RE for C, or use closure properties that we've already shown. You should write a few sentences to explain why your construction is correct, but you don't need to write a full, formal proof.

3. **(40 points):** Let $\#\mathtt{0}(w)$ be the number of `0`'s in w and $\#\mathtt{1}(w)$ be the number of `1`'s in w. For each language below, determine whether or not it is regular. Give a brief justification for each answer.

   (a) **(20 points):** $A_1 = \{w \in \{\mathtt{0},\mathtt{1}\}^* \mid \#\mathtt{0}(w) - \#\mathtt{1}(w)$ is divisible by $3\}$

   (b) **(20 points):** $A_2 = \{w \in \{\mathtt{0},\mathtt{1}\}^* \mid \#\mathtt{0}(w) - \#\mathtt{1}(w) < 3\}$

   (c) **(10 points, Extra Credit):** $A_3 = \left\{w \in \{\mathtt{0},\mathtt{1}\}^* \,\middle|\, |\#\mathtt{0}(w) - \#\mathtt{1}(w)| > 3\right\}$