

CPSC 314 Midterm

October 10, 2014

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

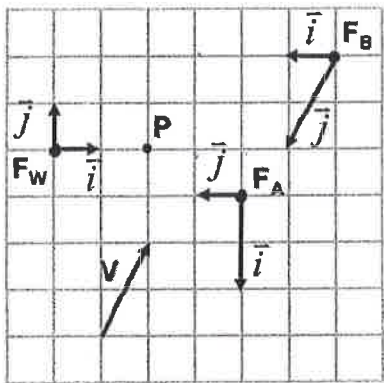
Name: Solutions

Student Number: _____

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TOTAL	/ 41

This exam has 6 questions, for a total of 41 points.

1. Coordinate Frames



$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_W = \begin{bmatrix} 0 & -1 & 4 \\ -2 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_A$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_A = \begin{bmatrix} 0 & 1 & -1.5 \\ 1 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_B$$

(a) (3 points) Express point P in each of the three coordinate frames.

$P_W(2,0)$

$P_A(-0.5,2)$

$P_B(3,1)$

(b) (3 points) Express point V in each of the three coordinate frames.

$V_W(1,2)$

$V_A(-1,-1)$

$V_B(0,-1)$

(c) (2 points) Find the 3×3 homogeneous transformation matrix which takes a point from F_A and expresses it in terms of F_W . I.e., determine M , where $P_W = MP_A$.

\Rightarrow need the origin and basis vectors of F_A , expressed in F_W .

$O_A = (4, -1)_W$
 $i_A = -2j_W$
 $j_A = -i_W$ } see matrix at top of page

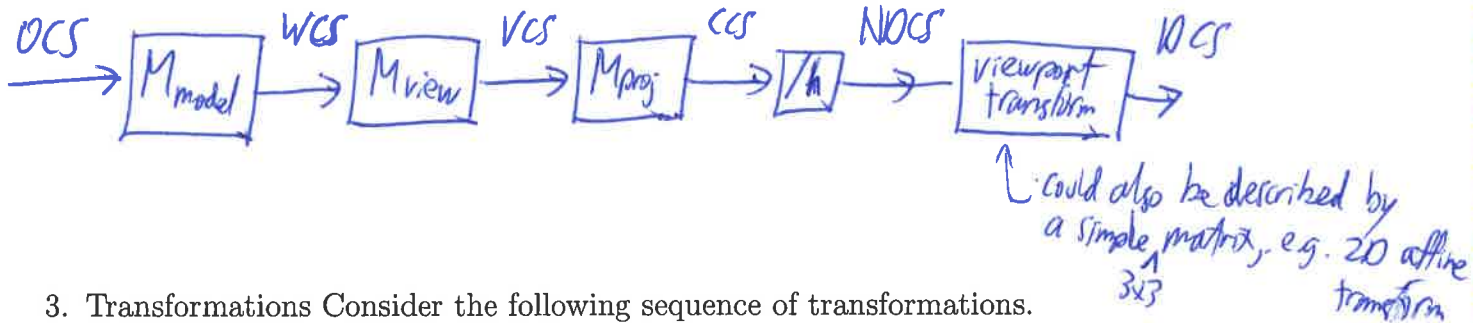
(d) (2 points) Find the 3×3 homogeneous transformation matrix which takes a point from F_B and expresses it in terms of F_A . I.e., determine M , where $P_A = MP_B$.

\Rightarrow need the origin and basis vectors of F_B expressed in F_A

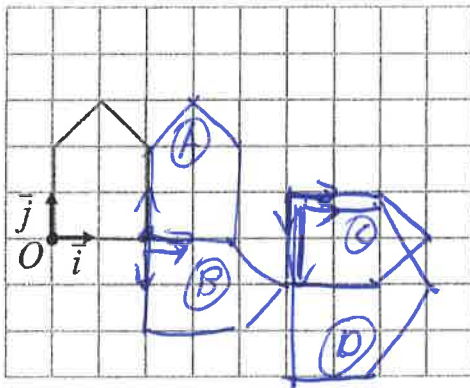
$O_B = (-1.5, -2)_A$
 $i_B = j_A$
 $j_B = i_A + j_A$ } see matrix at top of page
 ^
 2nd.

2. (5 points) Graphics Pipeline

Sketch a block diagram of the sequence of transformations in the graphics pipeline. Label the coordinate frames and the relevant transformation matrices. Abbreviations are fine.



3. Transformations Consider the following sequence of transformations.

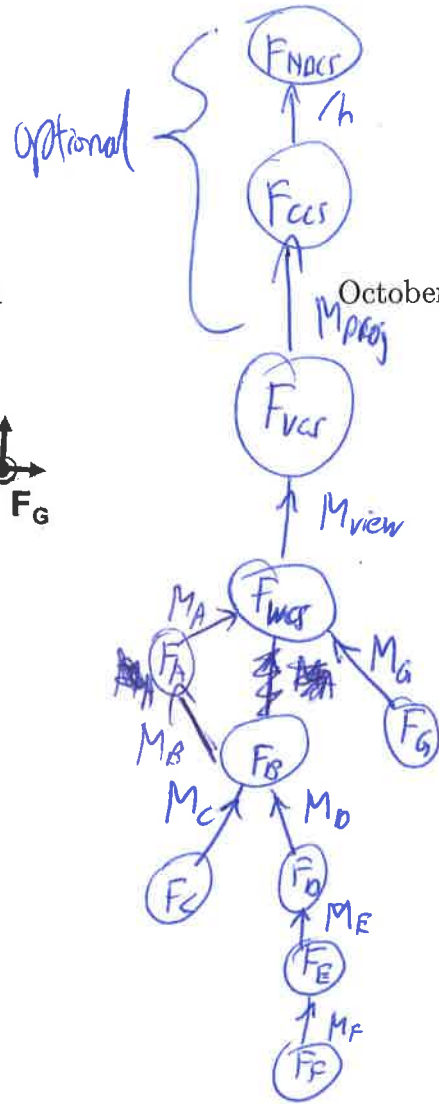
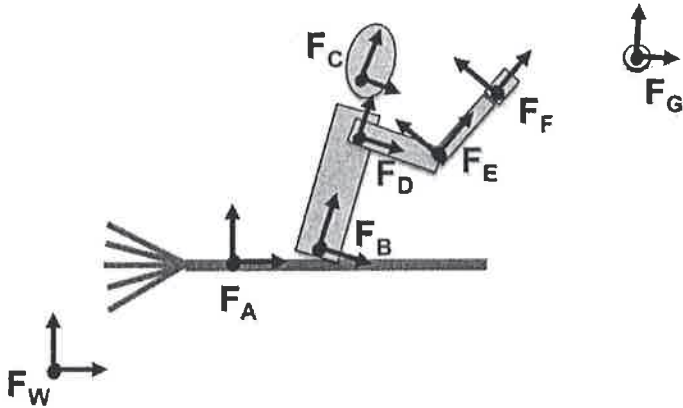


- A: $M.translate(2, 0, 0);$ ← drawHouse()
 - B: $M.rotate(-90, 0, 0, 1);$ ←
 - C: $M.translate(-1, 3, 0);$ ←
 - D: $M.scale(2, 1, 0);$ ←
- ↓ cumulative

- (a) (3 points) On the grid given above, sketch the house that results from the given sequence of calls to `drawHouse()`. Label these A, B, C, and D. Assume that the house as shown is drawn with the matrix M initialized to the identity matrix.
- (b) (1 point) If the individual transformation matrices are labeled $M_A, M_B, M_C,$ and $M_D,$ give an expression, in terms of these matrices, for the final compound transformation matrix, $M,$ that is produced by the above sequence.

$$M = M_A M_B M_C M_D$$

4. Scene Graphs



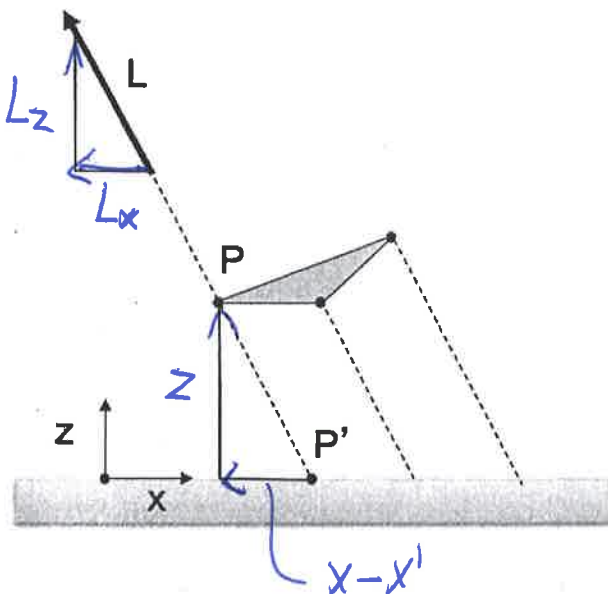
- (a) (3 points) Sketch the scene graph for the *Harry Potter* scene shown above, where he is reaching to catch the elusive *snitch* (ball). Label the transformations involved, using M_A to designate the matrix that takes points from coordinate frame F_A to its parent coordinate frame, etc. Include F_{VCS} , F_{WCS} , and M_{view} in your scene graph. Assume that the *snitch* and the broom are defined with respect to F_W , and that the torso, F_B , is defined with respect to the broom, F_A .
- (b) (2 points) Give an expression for the compound transformation that would indicate where the center of the *snitch* is located in terms of the hand coordinate frame, F_F .

$$P_F = M_F^{-1} M_E^{-1} M_D^{-1} M_C^{-1} M_G P_G$$

$$\hat{P}_G = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \text{ for centre of snitch}$$

5. Casting Shadows

A simple way to draw the shadow of a polygon is to use a transformation matrix M that computes the projected shadow location, $P'(x', y', z')$, for each polygon vertex $P(x, y, z)$ according to $P' = MP$. A dark-coloured "shadow polygon" can then be drawn using the projected vertices. As illustrated below for the xz -plane, a directional light source is defined by a vector, $L(L_x, L_y, L_z)$, that points in the direction of the light source.



Similar triangles (or, equivalently, slope):

$$\frac{L_z}{L_x} = \frac{z}{x - x'}$$

$$\Rightarrow x - x' = z \frac{L_x}{L_z}$$

$$x' = x - z \frac{L_x}{L_z}$$

Similarly, $y' = y - z \frac{L_y}{L_z}$

$$z' = 0$$

- (a) (5 points) Determine the projected shadow location, $P'(x', y', z')$, as a function of the point, $P(x, y, z)$, and the lighting direction, $L(L_x, L_y, L_z)$. Hint: First use similar triangles to develop an expression for x' . Similarly, develop expressions for y' and z' . Use the space above for your work.

- (b) (3 points) What is the name of the type of projection that is illustrated above? Give the transformation matrix, M , that would implement the projection, based on the expressions you developed in part (a).

This is a parallel projection.

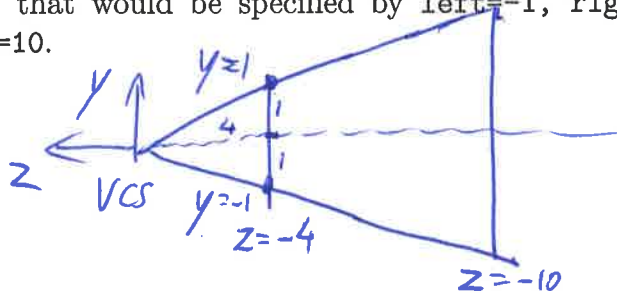
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{L_x}{L_z} \\ 0 & 1 - \frac{L_y}{L_z} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

6. Short Answer

- (a) (1 point) Express the point (6,4,3,2), given in homogenous coordinates, in cartesian coordinates.

$$\left(\frac{x}{h}, \frac{y}{h}, \frac{z}{h}\right) = (3, 2, 1.5)$$

- (b) (2 points) Sketch a side view, i.e., in the yz-plane, of the perspective view frustum that would be specified by left=-1, right=1, bot=-1, top=1, near=4, far=10.



- (c) (1 point) If the near value were doubled for the above example, what would the observed effect be on the resulting image?



The result would be a "zoom-in" by a factor of two on the center of the image.

- (d) (3 points) What do each of the following transformations matrices do? Give your answer in terms of scales, rotations, and translations. If it also implements a reflection, also indicate that (and show your work).

$$M = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

translate (-3, 7, 2)

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

scale (1, 2, 3)

$$M = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

could be a rotation, or reflection-and-rotation.
check: $i \times j = k$? No! therefore there is a reflection involved.

- (e) (2 points) Sketch where the VCS point $P(2, -1, -2)$ would appear in the final display window (draw this as a simple square) when used with the following projection matrix. Show your work.

$$\begin{bmatrix} 2 \\ -1 \\ 2/3 \\ 2 \end{bmatrix}_{VCS} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5/3 & -8/3 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ -2 \\ 1 \end{bmatrix}_{VCS}$$

$$\frac{1}{h} \Downarrow \begin{bmatrix} 1 \\ -0.5 \\ 1/3 \end{bmatrix}_{NDCS}$$

