Name: ______________________ **Student ID:** ______________________

You have 2.5 hours to complete the exam. Write your answers in the spaces provided.

No aids (books, notes, calculators, mobile phones, PDAs, telepathic transmitters, music players, other electronic devices, etc.) are permitted.

1) How can you test if a 2D point (x, y) is inside a triangle with vertices (x_0, y_0), (x_1, y_1) and (x_2, y_2)?

Evaluate the edge functions:

$$\begin{aligned} F_{01}(x,y) &= (x-x_0)(y-y_1)-(x-x_1)(y-y_0) \\ F_{12}(x,y) &= (x-x_1)(y-y_2)-(x-x_2)(y-y_1) \\ F_{20}(x,y) &= (x-x_2)(y-y_0)-(x-x_0)(y-y_2) \end{aligned}$$

The point is inside if and only if all of them have the same sign.

2) Take the barycentric coordinates (α, β, γ) of a point $\vec{x}$ with respect to a triangle with vertices $\vec{x}_0$, $\vec{x}_1$ and $\vec{x}_2$. If $\gamma < 0$, what side of the line through $\vec{x}_0$ and $\vec{x}_1$ is $\vec{x}$ on?

It is on the side opposite the point $\vec{x}_2$.

3) Give an explicit, parametric formula describing a sphere with centre $\vec{c}$ and radius r.

Use spherical coordinates:

$$\vec{x}(\theta,\phi) = r\,(\cos\theta\cos\phi, \quad \sin\phi, \quad \sin\theta\cos\phi) + \vec{c}$$

for $\theta \in [0, 2\pi]$ and $\phi \in [-\pi/2, \pi/2]$.

4) Describe the notion of field-of-view in terms of a view volume. (a diagram would help!)

It's the angle between sides of the view frustum extended back to the tip at the camera. *The diagram would be easier to see, but is a pain to produce electronically—sorry.*

5) How do we implement the perspective z-divide using homogenous coordinates?

Use a perspective projection matrix to map the camera z into the 4th homogeneous coordinate: for example

$$\begin{pmatrix} x \\ y \\ -z-1 \\ -z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

The homogenized version of the result (divided through by the fourth coordinate) will then have the right division:

$$\begin{pmatrix} -x/z \\ -y/z \\ 1+1/z \\ 1 \end{pmatrix}$$

6) Given a point with camera-space coordinates (x, y, z), and a perspective view frustum with the usual `left`, `right`, `bottom`, `top`, `near` and `far` bounds, work out where it maps to in an image with dimensions $M \times N$.

$$\left(\frac{M}{(\text{right}-\text{left})}\left[\frac{x}{z}(-\text{near})-\text{left}\right], \quad \frac{N}{(\text{top}-\text{bottom})}\left[\frac{y}{z}(-\text{near})-\text{bottom}\right], \right)$$

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7) Construct a model-view matrix for a camera at world-space location $(-2, 0, -2)$ looking at the world-space origin.

$$[\text{rotate}_\text{y}(135°)]\,[\text{translate}(2, 0, 2)]$$

8) How do you map a vertex normal under an affine transformation given by a 4×4 matrix M?

$$\begin{pmatrix} \vec{N}' \\ \cdot \end{pmatrix} = M^{-T} \begin{pmatrix} \hat{n} \\ 0 \end{pmatrix}$$
$$\hat{n}' = \frac{\vec{N}'}{\|\vec{N}'\|}$$

9) What is back-face culling, and how is it implemented at the rasterization stage of Z-buffer rendering?

It's where you do not render a triangle facing the wrong way from the camera (as if it was on the back side of a closed model). This is detected by requiring input triangles be specified with vertices in a right-handed way (with thumb point outward), and then not rasterizing any triangle oriented clockwise in the plane. The clockwise test can be implemented with the usual determinant for signed area (which is negative for clockwise oriented triangles).

10) Write down a general formula for describing a plane, and then a test to see if a ray with origin $\vec{x}_0$ and direction $\vec{d}$ intersects the plane.

Take a point $\vec{p}$ in the plane, and the normal $\hat{n}$ for the plane; it is described implicitly as all points $\vec{x}$ satisfying

$$(\vec{x} - \vec{p}) \cdot \hat{n} = 0$$

Plug the ray equation $\vec{x}(s) = \vec{x}_0 + s\vec{d}$ into this:

$$(\vec{x}_0 - \vec{p}) \cdot \hat{n} + s\vec{d} \cdot \hat{n} = 0$$

If $\vec{d} \cdot \hat{n} = 0$, there is no (unique) solution hence no intersection. Otherwise the root is:

$$s = \frac{-(\vec{x}_0 - \vec{p}) \cdot \hat{n}}{\vec{d} \cdot \hat{n}}$$

If this is non-negative, $s \geq 0$, it's an intersection.

11) Say you are given a BVH made of axis-aligned bounding boxes around a collection of spheres. Write efficient recursive pseudocode for checking a point $\vec{x}$ is inside any of the spheres.

Call this on the root node:

```
check(x, node):
if x is inside the bounding box of node:
  if node is a leaf:
    for all spheres contained here:
      if the distance from x to the centre is less than the radius:
        return true
  else:
    for all children:
```

```
      if check(x, child) is true:
        return true
return false
```

12) Describe a way that a BVH can be used to accelerate Z-buffer rendering.

Use it for view frustum culling: check the root node's box against the view frustum, and only recursively render its children if it overlaps.

13) Give a formula for computing the signed volume of a tetrahedron with vertices $\vec{x}_0$, $\vec{x}_1$, $\vec{x}_2$ and $\vec{x}_3$.

$$\frac{1}{6}\det\begin{pmatrix} x_0 & x_1 & x_2 & x_3 \\ y_0 & y_1 & y_2 & y_3 \\ z_0 & z_1 & z_2 & z_3 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

14) Name an effect produced by real physical cameras, and describe how path-tracing can capture it.

For example: depth-of-field. Sample a random 2D position on the finite aperture for each camera ray to go through.

15) How do you compute the gathered light at a point for diffuse (Lambertian) shading in a raytracer?

Add the ambient light to the contribution from each other light source, scaled by the cosine of the angle between the normal and the direction to the light source, and scaled by the visibility:

$$\vec{C}_{\text{gather}} = \vec{C}_{\text{ambient}} + \sum_{\text{light}\,i} \vec{C}_i (L_i \cdot \hat{n}) V_i$$

Here the visibility factor V_i is zero if the direction L_i to the light is opposite the surface normal $\hat{n}$ (if $L_i \cdot \hat{n} < 0$), and also is zero if a shadow ray cast to the light intersects any opaque geometry.

16) Can shadows be cast on mirror surfaces? Explain.

Yes—shadows are just a property of the visibility of the light source from a given point, and don't depend on the material of the surface. However, a shadow on a mirror won't look like a shadow on a diffuse surface.

17) What is the point of specifying surface normals at the vertices of a triangle mesh, where geometrically they are not well-defined?

It is a cheat to give the illusion of a smooth surface when shading it: fake shading normals can be smoothly interpolated from the vertex normals when calculating colours. The geometrically correct normals jump discontinuously across edges in the mesh.

18) Name and describe a shading effect that BRDF's cannot describe.

Sub-surface scattering, for example: here the light coming from a point on the surface may involve light that entered the surface elsewhere and then bounced around inside along some path. A BRDF only captures light reflected at the same surface point.

19) How do mipmaps deal with the minification problem?

They store a sequence of accurately resized versions of the original image, shrunk by powers of two all the way down to 1×1. When rendering a textured surface, you use the largest version of the image that can be magnified.

20) What is aliasing?

It is when a high frequency component appears as an incorrect low frequency component when minification is done without good filtering.

21) What should the colour stored in the 1×1 image in a mipmap be, in terms of the original image?

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It should be the average of all pixels in the original.

22) Why is biology involved in calibrating two different colour displays?

Two displays are said to be calibrated if the colours they display (for a given RGB value) appear the same to the human eye: the spectra they produce could be quite different as long as they appear as metamers to a human. Trying to match the spectra exactly (taking out the human biology from the equation) is pointless and probably impossible.