

Name: _____ Student ID: _____

Write your answers in the spaces provided.

No aids (books, notes, calculators, mobile phones, PDA's, music players, other electronic devices, etc.) are permitted.

1) Write down an explicit description as well as an implicit description of the line (in 2D) that passes through two distinct points $\vec{x}_0 = (x_0, y_0)$ and $\vec{x}_1 = (x_1, y_1)$.

An example explicit version: $\vec{x}(t) = \vec{x}_0 + t(\vec{x}_1 - \vec{x}_0)$ for parameter t .

Our standard implicit version: with $F(x, y) = (x_1 - x_0)(y - y_0) - (x - x_0)(y_1 - y_0)$, the points where $F(x, y) = 0$.

2) Determine a bounding box for a circle with centre $(2, 5)$ and radius 1.

A tight-fitting box is $[1, 3] \times [4, 6]$.

3) Given a triangle with vertices (x_0, y_0) , (x_1, y_1) and (x_2, y_2) and the usual edge functions such as $F_{01}(x, y) = (x_1 - x_0)(y - y_0) - (x - x_0)(y_1 - y_0)$, detail how to compute the barycentric coordinates of a point (x, y) . In particular, identify α as the barycentric coordinate corresponding to vertex 0, β for vertex 1, and γ for vertex 2.

Let $S = F_{01}(x, y) + F_{12}(x, y) + F_{20}(x, y)$. Then

$$\alpha = \frac{F_{12}(x, y)}{S}$$

$$\beta = \frac{F_{20}(x, y)}{S}$$

$$\gamma = \frac{F_{01}(x, y)}{S}$$

4) How can you construct a right-handed orthonormal 3D basis with a given unit-length \vec{r} as its first vector?

First find a vector \vec{u} not parallel to \vec{r} . If $\vec{r} = (\pm 1, 0, 0)$ then take $\vec{u} = (0, 1, 0)$ and otherwise $\vec{u} = (1, 0, 0)$. Then

$$\vec{s} = \frac{\vec{r} \times \vec{u}}{\|\vec{r} \times \vec{u}\|}$$

and

$$\vec{t} = \vec{r} \times \vec{s}$$

5) Determine a model-view transformation corresponding to a camera with world space coordinates $(5, 0, 5)$, pointing at the origin of world space. You can express this as a sequence of named transformations (e.g. rotate around this axis by this angle) or if you prefer as a 4×4 matrix.

First translate by $(-5, 0, -5)$ to move the camera to the origin, then rotate around the y -axis by -45° to have it pointing along the $-z$ axis in camera space.

6) The projection transformation maps points from which coordinates system to which other coordinate system?

From camera space to normalized device coordinates.

7) What is the difference between the view volume for an orthographic and a perspective projection?

The orthographic view volume has parallel sides; the perspective view volume is a frustum instead.

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8) Why is it important to specify a near clipping plane for a perspective projection?

Many reasons, including the fact that points behind the camera $z > 0$ are mapped in front of the camera, if they haven't been clipped.

9) How is the z -divide of perspective projection attained using matrix multiplication?

Using homogeneous coordinates, the matrix can be set up with bottom row something like $(0, 0, -1, 0)$, so that the output homogenous coordinate is proportional to the input z coordinate. Then homogenization divides by the output homoegeous coordinate, which is proportional to z .

10) Write recursive pseudo-code for rendering a hierarchical model.

To render a node:

- *Push the current model-view matrix onto the stack.*
- *Multiply the model-view matrix by this node's transformation from its object space to its parent's.*
- *Render any geometry at this node.*
- *Render any children recursively.*
- *Pop the top from the matrix stack, returning the model-view matrix to what it was before.*

11) Sketch a configuration of triangles for which the painter's algorithm fails. How can it be fixed?

Any two triangles which intersect each other, or triangles woven together like blades, would provide an example. It can be resolved, for example, by splitting triangles up into smaller triangles that can be depth sorted, or going to the Z-buffer algorithm instead.