CPSC 121 Midterm 2
Monday November 14th, 2016

Name: ______________________ Student ID: ______________________

Signature: ______________________ Section (circle one): 11:00 15:30 17:00

Question	Marks
1	
2	
3	
4	
5	
Total	

– You have 75 minutes to write the 5 questions on this examination.
A total of 55 marks are available.

– **Justify all of your answers.**

– You are allowed to bring in one hand-written, double-sided 8.5 x 11in sheet of notes, and nothing else.

– Keep your answers short. If you run out of space for a question, you have written too much.

– The number in square brackets to the left of the question number indicates the number of marks allocated for that question. Use these to help you determine how much time you should spend on each question.

– Use the back of the pages for your rough work.

– **Good luck!**

UNIVERSITY REGULATIONS:

– Each candidate should be prepared to produce, upon request, his/her UBC card.

– No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

– CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
2. Speaking or communicating with other candidates.
3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

– Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

[15] 1. Consider the theorem

For any integer n, if $2^n - 1$ is a prime, then n is also prime.

[3] a. Translate the theorem statement into predicate logic. You can use the predicate *Prime*(x) which is true when x is a prime.

[4] b. Suppose that you decide to prove this theorem using a **direct proof**. Write down what you would assume and what you would need to show. You can use the predicate *Prime*(x). **Do not prove the theorem.**

[4] c. Suppose that you decide to prove this theorem using a **proof by contrapositive**. Write down what you would assume and what you would need to show. You can use the predicate *Prime*(x). **Do not prove the theorem.**

[4] d. Suppose that you decide to prove this theorem using a **proof by contradiction**. Write down what you would assume and what you would need to show. You can use the predicate *Prime*(x). **Do not prove the theorem.**

[15] 2. Your friend designed an algorithm whose execution requires $4n^3 + 2n^2$ steps where n is the size of the input.

Hint: Suppose that an algorithm runs in $f(n)$ steps where n is the size of the input. Recall that the number of steps of this algorithm is in $O(g)$ if the following proposition is true:

$$\exists c \in \mathbf{R}^+ \, \exists n_0 \in \mathbf{N} \, \forall n \in \mathbf{N}, n \geq n_0 \rightarrow f(n) \leq cg(n). \qquad (*)$$

[6] a. Prove that the number of steps of your friend's algorithm is in $O(n^4)$.

[3] b. Negate the proposition in $(*)$ and bring the negation all the way to the right so that there is no negation in front of any quantifier.

[6] c. **Using the proposition in part b**, prove that the number of steps of your friend's algorithm is **NOT** in $O(n^2)$.

[10] 3. Consider the following theorem:

For any integers a, b and c, if $a^2 + b^2 = c^2$, then at least one of a and b is even. $(*)$

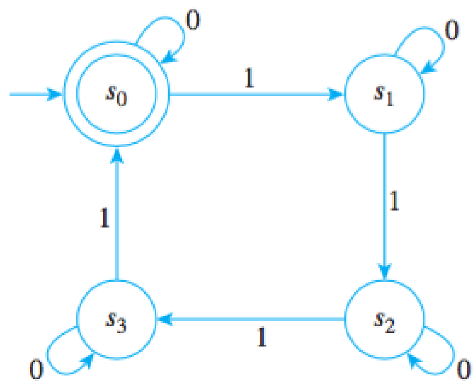
To guide you through proving this theorem, we broke down the proof into 3 steps below.

[3] a. First, prove that for any integers a, b and c, if $a^2 + b^2 = c^2$ and a and b are both odd, then c^2 is even.

[3] b. Second, prove that for any integer c, if c^2 is even, then c^2 is divisible by 4.
Hint: we proved in class that if the square n^2 of an integer n is even, then n is even.

[4] c. Prove theorem $(*)$ using the results from part (a) and (b) above.
Hint 1: a **proof by contradiction** works well here.
Hint 2: it may be useful to show that $(a^2 + b^2)$ is not divisible by 4.

[9] 4. Consider the following deterministic finite-state automaton. Assume that every input is a string of 0's and 1's.



[4] a. Which of the following words will this finite-state automaton accept?
Circle one of Yes/No for each string.

- **01010** Yes No
- **11001** Yes No
- **110110** Yes No
- **101111** Yes No
- **1101101** Yes No
- **111101100** Yes No
- **110111011** Yes No
- **111101111** Yes No

[3] b. Describe as simply as possible the set of inputs that will lead you to each state.

[2] c. Describe as simply as you can the set of strings that this finite-state automaton accepts.

[6] 5. Convert the DFA to a sequential circuit. We have given you several components of the sequential circuit below. Please fill in the missing parts.