CPSC 121 Midterm 1
Friday October 14th, 2016

Name: ______________________ Student ID: ______________________

Signature: ______________________ Section (circle one): 11:00 15:30 17:00

– You have 70 minutes to write the 9 questions on this examination. A total of 60 marks are available.

– **Justify all of your answers.**

– You are allowed to bring in one hand-written, double-sided 8.5 x 11in sheet of notes, and nothing else.

– Keep your answers short. If you run out of space for a question, you have written too much.

– The number in square brackets to the left of the question number indicates the number of marks allocated for that question. Use these to help you determine how much time you should spend on each question.

– Use the back of the pages for your rough work.

– **Good luck!**

Question	Marks
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

UNIVERSITY REGULATIONS:

– Each candidate should be prepared to produce, upon request, his/her UBC card.

– No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

– CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
2. Speaking or communicating with other candidates.
3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

– Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

[12] 1. [1] a. For what truth values of a and b is $a \to b$ false?

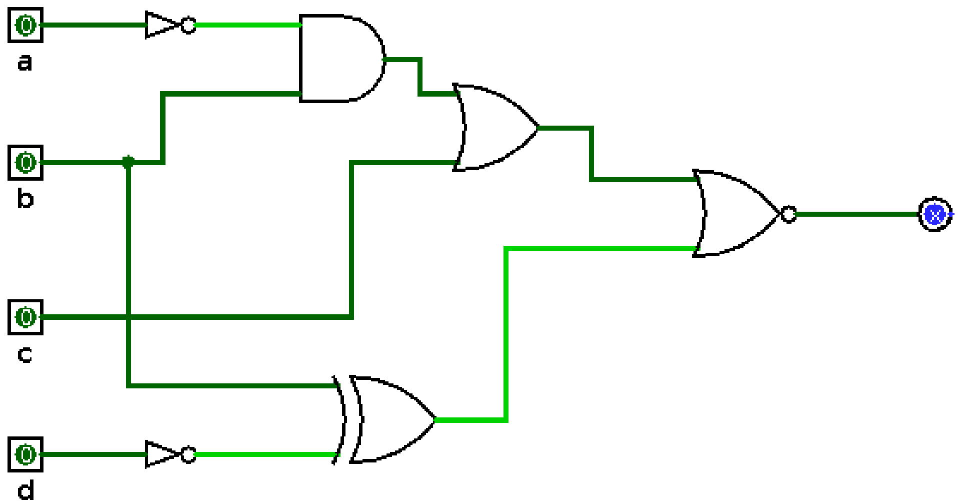
[3] b. Is $(a \oplus b) \leftrightarrow (a \wedge \sim b) \vee (\sim a \wedge b)$ a tautology, a contradiction, or a contingency? Justify your answer.

[8] c. Using a sequence of logical equivalence, prove that

$$(p \oplus q) \vee \sim(\sim p \to q) \equiv \sim p \vee \sim q$$

Please write the name of the law(s) you applied at each step. Hint: What you showed in part b above may be helpful.

[6] 2. [3] a. Consider the following digital circuit. Write a propositional logic expression which is the direct translation of this circuit to propositional logic. Do not simplify your expression.



[3] b. Draw the circuit corresponding to the following propositional logic expression.

$$(a \oplus (\sim b \wedge c)) \vee \sim(a \wedge d)$$

[10] 3. Assume that we use **6 bits** to represented binary integers. If the binary integer is interpreted as a signed integer, then we will represent it using two's complement.

[3] a. Consider the 6-bit binary integer **110101**. If we interpret it as a **signed** binary integer, what is the corresponding decimal value? If we interpret it as an **unsigned** binary integer, what is the corresponding decimal value?

[3] b. Consider the hexadecimal value **1F**. What is the corresponding binary value? What is the corresponding decimal value?

[1] c. Write down the two binary integers from parts (a) and (b). Then add them together in binary. What is the resulting binary integer (assuming that we only have 6 bits to represent it)? Please write down your answer in this form A + B = C where A, B, and C are binary numbers. For example, this can be a possible answer: 001000 + 101010 = 111100.

[1] d. Consider your answer A + B = C from part (c). Suppose that we interpret all three binary integers A, B, and C as unsigned binary integers. Convert all three binary integers into decimal values and write down the same addition in decimal. For example, this can be a possible answer: 30 + 60 = 120.

[2] e. Does your answer from part (d) make sense? Why or why not?

[8] 4. Determine the validity of the following argument *using rules of inference and/or logical equivalences*. You must state what inference rule or logical equivalence you use at each step. Do not rewrite the premises; start your proof at step 6.

1. $p \oplus q$
2. $(s \rightarrow r) \rightarrow t$
3. $(t \rightarrow m) \vee u$
4. $r \vee \sim q$
5. $\sim u \wedge \sim m$

$\therefore\ p \wedge s$

[3] 5. You were asked to prove that an argument such as the following is valid:

1. *premise* 1
2. *premise* 2
3. *premise* 3

$\therefore\ p \wedge s$

but instead of proving $p \wedge s$ you succeeded in proving $\sim(p \wedge s)$. A teaching assistant verified your proof and confirmed (correctly!) that you did not make a mistake. What are the two possible reasons for the perplexing conclusion you reached?

[6] 6. Consider the following definitions:

- P: the set of all people living around 200 BC.
- T: the set of tools/weapons available at that time.
- $G(x)$: person x is Greek
- $R(x)$: person x is Roman
- $W(x, y)$: person x wielded weapon y
- $\mathit{Talked}(x, y)$: person x talked to person y
- $\mathit{Fought}(x, y)$: person x fought person y

Translate each of the following English statements into predicate logic. For instance, "Alex wielded some weapon" would be translated as $\exists x \in T, W(\text{Alex}, x)$.

[3] a. A Greek wielding a bronze sword (an element of T) fought a Roman wielding an iron shield (another element of T).

[3] b. People who used cudgels (an element of T) fought every Greek they talked to.

[3] 7. Using the same definitions in the previous question, translate the following predicate logic statements into English:

$$\forall x \in P, (G(x) \wedge \exists y \in P, R(y) \wedge \mathit{Talked}(x, y)) \rightarrow (\exists z \in P, G(z) \wedge x \neq z \wedge \mathit{Talked}(x, z))$$

[3] 8. Why do we almost never translate an English sentence into a predicate logic statement of the form $\exists x \in D, P(x) \rightarrow Q(x)$?

[9] 9. In this question, you will design a circuit that takes as input a 4-bit unsigned binary integer $x_3x_2x_1x_0$, and outputs its integer square root as a 2-bit unsigned binary integer y_1y_0. By "integer square root", we mean that any fractional part is discarded. For instance, the integer square root of 11 is 3 because $\sqrt{11} = 3.3166\ldots$

a. Write a proposition for the value of y_1. Any correct proposition will be worth at least 1.5/2. In order to get 2/2, you need to write one that is not too ugly. Hint: our solution is very short.

b. Write a proposition for the value of y_0. Any correct proposition will be worth at least 2.5/4. In order to get 4/4, you need to write one that is not too ugly.

c. Finally draw your circuit below.