CPSC 121 Midterm 2
Tuesday March 15th, 2016

Name: _____ Student ID: _____

Signature: _____ Section (circle one): Morning Afternoon

- You have 70 minutes to write the 6 questions on this examination. A total of 50 marks are available.
- **Justify all of your answers.**
- You are allowed to bring in one hand-written, double-sided 8.5 x 11in sheet of notes, and nothing else.
- Keep your answers short. If you run out of space for a question, you have written too much.
- The number in square brackets to the left of the question number indicates the number of marks allocated for that question. Use these to help you determine how much time you should spend on each question.
- Use the back of the pages for your rough work.
- **Good luck!**

Question	Marks
1	
2	
3	
4	
5	
6	
Total	

UNIVERSITY REGULATIONS:

- Each candidate should be prepared to produce, upon request, his/her UBC card.
- No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.
- CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
    1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
    2. Speaking or communicating with other candidates.
    3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

[10] 1. For each of the following statements, write down ALL of the proof strategies (from the list given below), that are appropriate to use to prove the statement. Just write down the LETTER for each strategy (do not write the full name):

A. Constructive or non-constructive direct proof of existence.

B. Exhaustive proof.

C. Direct proof by generalizing from a generic particular (i,e, let x be any non particular (unspecified)...).

D. Proof by cases.

E. Contrapositive proof.

F. Antecedent assumption (called direct proof by Epp).

G. Proof by contradiction.

Note that you must **NOT** prove the statement. You just need to suggest all the proof strategies which can be used to prove it. In some cases, translating the statement into predicate logic first might be useful (although it is not required).

[2] a. Every CPSC 121 student who plays tennis knows a student who enjoys swimming.

[2] b. If an integer x can be written as $x = 5y + 7z$ for some integers y and z, then $x + 1$ can be written as $x + 1 = 5y' + 7z'$ for some integers y' and z'.

[2] c. An hexagon (6-sided polygon) can not have more than 3 angles that measure more than 200° each.

[2] d. At least four UBC Computer Science instructors went to the 46^{th} ACM Symposium on Computer Science Education, held from March 2^{nd} to March 5^{th} 2016 in Memphis, Tennessee.

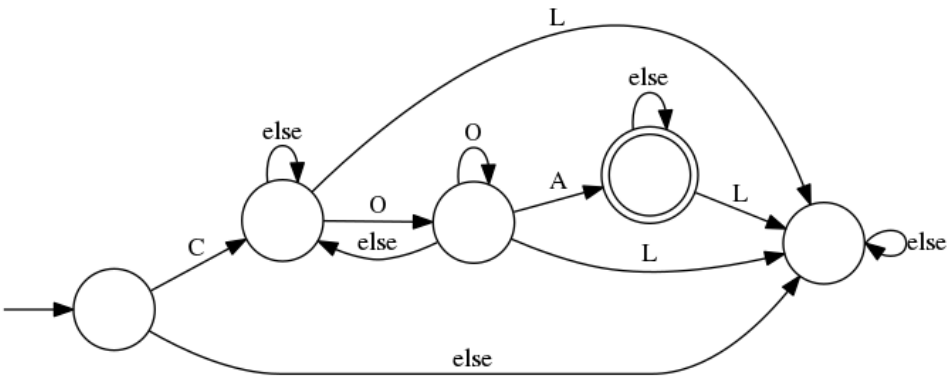
[2] e. Every positive integer larger than 1 can be written as a sum of one or more, not necessarily distinct, primes.

[5] 2. Provide a formal proof (by using one formal rule of inference at a time and explicitly stating the rule that is used on the right side) for the following argument. You don't need to rewrite the premises in the proof. Just continue with step 3.

1. $\exists x \in D, \forall y \in D, \sim Q(x, y)$
2. $\forall x \in D, \exists y \in D, \sim P(x) \rightarrow Q(x, y)$

$\therefore \exists x \in D, P(x)$

[7] 3. Consider the following deterministic finite-state automaton:



[4] a. Which of the following words will this finite-state automaton accept (circle one of Yes/No for each word)?

- **CROSSROAD** Yes No
- **COLGATE** Yes No
- **BLOOD** Yes No
- **COCOON** Yes No
- **LOATH** Yes No
- **COCOA** Yes No
- **OATMEAL** Yes No
- **COAXAL** Yes No

[3] b. Describe as simply as you can the set of words that this finite-state automaton accepts.

[5] 4. Prove the following logical argument **using a proof by contradiction**:

1. $p \vee r$
2. $p \rightarrow q$
3. $r \rightarrow t$
4. $\sim(t \vee u)$

$\therefore\ q$

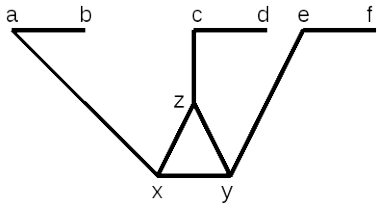
[16] 5. Prove each of the three following theorems using a proof technique of your choice.

a. Let us say that an integer x is *almost divisible* by an integer y if dividing x by y leaves a remainder that is either 1 or $y - 1$. For instance, both 15 and 20 are almost divisible by 7.

Theorem: If an integer x is almost divisible by an integer y, then dividing x^2 by y always leaves a remainder of 1.

b. **Theorem**: There are primes whose product is one less than another prime.

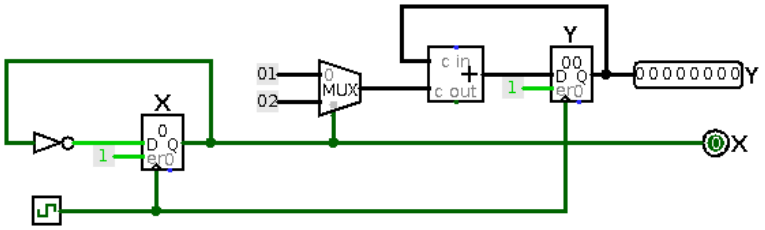
c. Suppose that we have an art gallery that is shaped (rather strangely) as follows:



Each line segment is a corridor; corners are labeled a, b, c, d, e, f, x, y and z. We want to place guards at some of the corners, in order to guard the art work displayed in each of the corridors, and we'd like to use as few guards as possible (they're expensive). Observe that either corner a or corner b must have a guard (otherwise the corridor joining them isn't guarded). Similarly, either corner c or corner d must have a guard, and either corner e or corner f must have a guard.

Theorem: If we placed a guard at at least one of corners a, c and e, then we will only need to place guards at TWO of corners x, y and z.

[7] 6. Consider the following sequential circuit:



Please observe that the registers labeled X and Y initially contain the value 0.

[3] a. Fill in the following table indicating the contents of registers X and Y after 1, 2, 3, 4 and 5 clock cycles.

Clock cycles	X	Y
1		
2		
3		
4		
5		

[4] b. The following sequential circuit will output 1 if the sequence of bits it receives as input contains three **consecutive** 1 bits. Unfortunately the instructor designing the circuit made **two mistakes**, and **left out a small part of the circuit**. Fix the mistakes and add the part of the circuit that was left out.