The University of British Columbia

Computer Science 121

Midterm 1

October 9, 2015

Circle your Section:

Section 101, Instructor: George Tsiknis

Section 102, Instructor: Patrice Belleville

Time: 70 minutes

Total marks: 60

Name____Student No______________
(PRINT) (Last) (First)

Signature____

<u>This examination has 8 pages. Check that you have a complete paper.</u>

This is a closed book exam. You may use the formula sheet which you have brought with you, but no other books or notes. You cannot use a calculator.

Answer all the questions on this paper.

Give very **short but precise** answers. Always use point form where it is appropriate.

Work fast and do the easy questions first. Leave some time to review your exam at the end.

The marks for each question are given in []. Use this to manage your time.

Good Luck

MARKS	
1	
2	
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Total	

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1. [4] Is the following statement is a tautology, a contradiction or neither? Justify your answer using a method of your choice.

$$((p \rightarrow q) \wedge \sim p) \rightarrow \sim q$$

2. [6] Translate the following sentences into propositional logic using the following variables with the given meaning:

w = the weather improves
g = we run out of gas
d = we will get to our destination
t = we will be late

a. **Unless the weather does not improve or we run out of gas, we will get to our destination, but we will be late. (**recall **p unless q** means **if ~q then p**)

b. **Assuming that we will not run out of gas, if the weather improves, we will get to our destination but we will be late.**

3. [5] Write a **short** propositional logic expression that corresponds to the following truth table (i.e. defines the value of s based on p, q and r):

p	**q**	**r**	**s**
F	F	F	T
F	F	T	T
F	T	F	F
F	T	T	T
T	F	F	F
T	F	T	F
T	T	F	F
T	T	T	T

Expression:

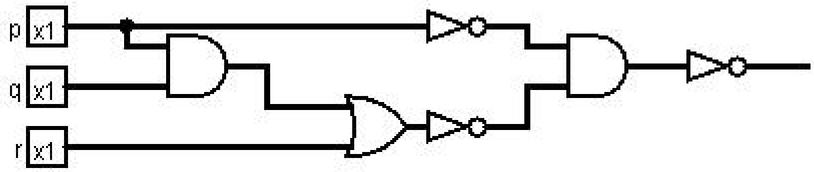
4. **[7]** Prove the following equivalence:

$$(p \land \sim q) \rightarrow \sim p \equiv p \rightarrow q$$

Use a **formal logical equivalence proof** showing the equivalence rule that is applied at each step.

Proof: **rule applied**

5. [8] Consider the following digital circuit:



This circuit is input/output equivalent to another one that uses exactly one logical gate with two inputs. Show the simpler circuit by answering the following:

a. [2] Write a propositional logic expression which is the direct translation of this circuit to propositional logic. Do not simplify your expression.

b. [4] Using a sequence of logical equivalences (showing the rules as well), simplify this expression to one that has only two propositional variables.

c. [2] Finally draw the circuit corresponding to the simplified expression.

6. [8] Number representation

The following table shows the first few powers of 2, and may be helpful for this question:

power	value
2^0	1
2^1	2
2^2	4
2^3	8
2^4	16
2^5	32

power	value
2^6	64
2^7	128
2^8	256
2^9	512
2^{10}	1,024
2^{11}	2,048

power	value
2^{12}	4,096
2^{13}	8,192
2^{14}	16,384
2^{15}	32,768
2^{16}	65,536
2^{20}	1,048,576

a. [2] What is the binary representation of the (signed) decimal value 120 using **8 bits**?

b. [2] What decimal value is represented by the **6-bit unsigned** binary number 110110?

c. [2] What decimal value is represented by the 6-bit **signed** binary number 110110?

d. [2] What is the binary representation of the (signed) decimal value -64 using **8 bits**?

7. [8] Circuit Design 1

Design a 3-inputs minority circuit. This circuit has three inputs x, y and z, and should output True if at most one of x, y, z is true. The circuit should output False if two or more of x, y, z are true.

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8. [6] Consider the following propositional logic proof. At each step indicate the missing part, which is either the conclusion of a rule or the rule that is used for the step (and the statements the rule is applied to) .

1.	$p \vee q$	**premise**
2.	$q \rightarrow r$	**premise**
3.	$p \wedge u \rightarrow t$	**premise**
4.	$\sim q \rightarrow (w \wedge u)$	**premise**
5.	$\sim r$	**premise**
6.	$\sim q$	________________
7.	________________	**1, 6, elimination**
8.	________________	________________
9.	u	**8, specialization**
10.	________________	________________
11.	t	**3, 10, modus ponens**

9.[8] Consider the following definitions:

- PM: the set of all Canadian prime ministers.
- C(x): x is/was a member of the Conservative Party.
- F(x): x likes fishing
- L(x): x is/was a member of the Liberal Party.
- W(x): x is a woman.

a) Translate each of the following English statements into predicate logic.

i. Every Canadian prime minister is/was member of either the Liberal or the Conservative Party.

ii. Every woman who is/was prime minister of Canada is/was a member of the conservative Party.

b) Translate each of the following predicate logic statements into English.

i. $\sim (\exists \mathbf{x} \in \mathrm{PM}, L(x) \wedge C(x))$

ii. $\forall \mathbf{x} \in \mathrm{PM}, (\sim W(x) \wedge L(x)) \rightarrow \sim F(x)$