

Computer Science 121
Midterm 1 Solutions

1. A truth table for the statement:

p	q	$p \rightarrow q$	$\sim p$	$(p \rightarrow q) \wedge \sim p$	$\sim q$	whole statement
F	F	T	T	T	T	T
F	T	T	T	T	F	F
T	F	F	F	F	T	T
T	T	T	F	F	F	T

Therefore the statement is neither a tautology nor a contradiction.

2.

a. $\sim(\sim w \vee g) \rightarrow (d \wedge t)$ OR $(w \wedge \sim g) \rightarrow (d \wedge t)$

If we interpret “but we will be late” as “in any case we will be late”:

$$(\sim(\sim w \vee g) \rightarrow d) \wedge t \quad \text{OR} \quad ((w \wedge \sim g) \rightarrow d) \wedge t$$

b. $\sim g \rightarrow (w \rightarrow (d \wedge t))$

OR

$$(\sim g \wedge w) \rightarrow (d \wedge t)$$

3.

Expression:

$$(\sim p \wedge \sim q) \vee (q \wedge r)$$

OR

$$\sim((q \wedge \sim r) \vee (p \wedge \sim q))$$

4.

Proof:

$(p \wedge \sim q) \rightarrow \sim p$	given
$\equiv \sim(p \wedge \sim q) \vee \sim p$	definition of \rightarrow
$\equiv (\sim p \vee \sim \sim q) \vee \sim p$	DeMorgan
$\equiv (\sim p \vee q) \vee \sim p$	double negation (may skip)
$\equiv \sim p \vee (\sim p \vee q)$	commutative (may skip)
$\equiv (\sim p \vee \sim p) \vee q$	associative
$\equiv \sim p \vee q$	idempotence
$\equiv p \rightarrow q$	definition of \rightarrow

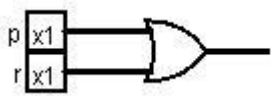
5.

a. $\sim[\sim p \wedge \sim((p \wedge q) \vee r)]$

b.

$$\begin{aligned} &\sim[\sim p \wedge \sim((p \wedge q) \vee r)] \\ \equiv &\sim\sim p \vee \sim\sim((p \wedge q) \vee r) && \text{DM} \\ \equiv &p \vee ((p \wedge q) \vee r) && \text{DNEG (twice)} \\ \equiv &(p \vee (p \wedge q)) \vee r && \text{ASS} \\ \equiv &p \vee r && \text{ABS} \end{aligned}$$

c. The circuit can be implemented with a single OR gate:



6.

a. 0 1111000

b. 54

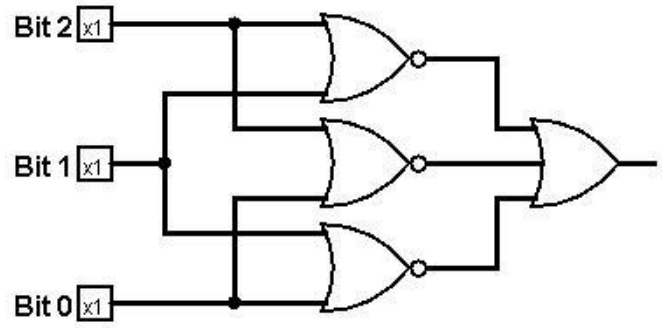
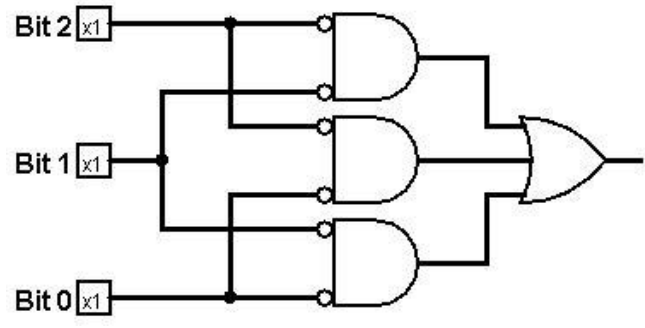
c. -10

d.

	01000000	64
	10111111	
+1	00000001	
	11000000	-64

7.

OR



8.

1.	$p \vee q$	premise
2.	$q \rightarrow r$	premise
3.	$p \wedge u \rightarrow t$	premise
4.	$\sim q \rightarrow (w \wedge u)$	premise
5.	$\sim r$	premise
6.	$\sim q$	<u>2, 5, modus tollens</u>
7.	<u>p</u>	1, 6, elimination
8.	<u>$w \wedge u$</u>	<u>4, 6, modus ponens</u>
9.	u	8, specialization
10.	<u>$p \wedge u$</u>	<u>7, 9, conjunction</u>
11.	t	3, 10, modus ponens

9. a)

i. $\forall p \in \text{PM}, L(p) \vee C(p)$

OR

$\forall p \in \text{PM}, L(p) \oplus C(p)$

ii. $\forall p \in \text{PM}, W(p) \rightarrow C(p)$

b)

- i. No prime minister of Canada is/was a member of the Liberal and the Conservative Party
- ii. Every prime minister of Canada who is not /was not a woman and is/was a member of the Liberal Party, does not like fishing

OR (assuming that not female means male):

Every male prime minister of Canada who is not /was a member of the Liberal Party, does not like fishing