CPSC 121 Midterm 1
Friday February 5th, 2016

Name: ____________________ Student ID: ____________________

Signature: ____________________ Section (circle one): Morning Afternoon

– You have 70 minutes to write the 8 questions on this examination. A total of 60 marks are available.

– **Justify all of your answers.**

– You are allowed to bring in one hand-written, double-sided 8.5 x 11in sheet of notes, and nothing else.

– Keep your answers short. If you run out of space for a question, you have written too much.

– The number in square brackets to the left of the question number indicates the number of marks allocated for that question. Use these to help you determine how much time you should spend on each question.

– Use the back of the pages for your rough work.

– **Good luck!**

Question	Marks
1	
2	
3	
4	
5	
6	
7	
8	
Total	

UNIVERSITY REGULATIONS:

– Each candidate should be prepared to produce, upon request, his/her UBC card.

– No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

– CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
2. Speaking or communicating with other candidates.
3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

– Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

[3] 1. Is the following statement a tautology, a contradiction or neither? Justify your answer using a truth table:

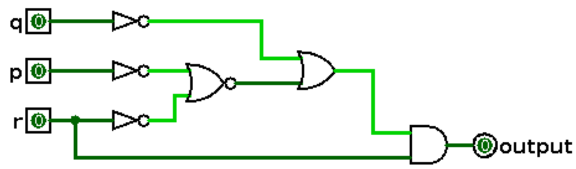
$$(p \leftrightarrow q) \rightarrow (p \oplus \sim q)$$

[6] 2. Using a sequence of logical equivalences, prove that

$$(\sim p \wedge \sim q) \vee \sim(\sim(r \wedge q) \rightarrow p) \equiv \sim p \wedge (\sim q \vee \sim r)$$

Write the name of the law you applied at each step.

[9] 3. Consider the following digital circuit:



[3] a. Write a propositional logic expression which is the direct translation of this circuit to propositional logic. Do not simplify your expression.

[4] b. Simplify your answer from part (a) using a sequence of logical equivalences. Write the name of the law you applied at each step.

[2] c. Draw the simplified circuit corresponding to your answer for part (b). Hint: you should only need 3 gates.

[11] 4. Assume that we use 6 bits to represent signed binary integers (using two's complement).

[3] a. What is the 6-bit binary representation of the signed decimal value -15? What is the corresponding hexadecimal representation using 2 digits?

[3] b. What is the 6-bit binary representation of the hexadecimal value 2E? What is the corresponding signed decimal value?

[1] c. With 6 bits, what is the smallest (most negative) signed binary integer that you can represent?

[2] d. What is the sum of the two 6-bit signed binary integers from parts (a) and (b)? What is the corresponding decimal value?

[2] e. Why is your decimal answer from part (d) positive?

[12] 5. Three mafiosi (Don Cortizone, Don Maledictione and Don Sanconvictione) went to the horse races. Inspector Montalbano, who is watching them, learns that

- If Don Sanconvictione bet on Gallant Fox in the third race, then Don Maledictione paid Secretariat's jockey to lose.
- If Don Cortizone provided EPO to Whirlaway, then Don Sanconvictione and Don Maledictione bet on Count Fleet in the seventh race.
- Either Don Maledictione did not win any of his bets, or Don Sanconvictione was responsible for War Admiral's sickness (or both).
- If Don Sanconvictione and Don Maledictione bet on Count Fleet in the seventh race and Don Sanconvictione was responsible for War Admiral's sickness, then Don Maledictione did not pay Secretariat's jockey to lose.

After the races were over, Detective Catarella brought him proof that not only Don Cortizone provided EPO to Whirlaway, but also that Don Maledictione won all five of his bets.

[3] (a) Rewrite each of these statements using propositional logic. Make sure to define the propositions you are using by underlining part of the sentence above (for instance "Don Sanconvictione bet on Gallant Fox in the third race") and putting the letter (for instance, "g") next to it.

[9] (b) Using your answer from part (a), known logical equivalences and the rules of inference, prove that Don Sanconvictione did not bet on Gallant Fox in the third race. Write the name of the equivalence law or the inference rule you applied at each step.

[6] 6. Consider the following definitions:

- C: the set of all colors.
- S: the set of all shoppers.
- I: the set of all items (dress, shoes, pants, hats, gloves, shirts, etc).
- *Knows*(x, y): shopper x knows shopper y.
- *Bought*(x, y, z): shopper x bought item y in color z.

translate each of the following English statements into predicate logic. For instance, "Mrs. Nelly bought a colored skirt" would be translated as $\exists x \in C, \textit{Bought}(\text{Mrs. Nelly}, \text{skirt}, x)$.

[3] a. A shopper who knows Cédric bought a blue shirt.

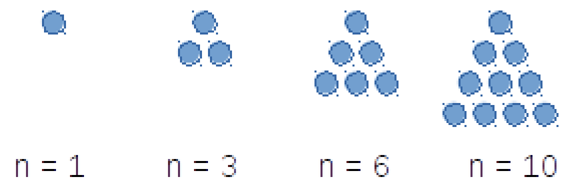
[3] b. Every shopper who bought green gloves knows a shopper who bought a red hat.

[6] 7. Using the same definitions as for the previous question, translate each of the following predicate logic statements into English.

[3] a. $\forall x \in I, \forall y \in C, \textit{Bought}(\text{Nicolas}, x, y) \rightarrow \textit{Bought}(\text{Chen}, x, y)$.

[3] b. $\exists x \in S, \forall y \in S, \textit{Bought}(y, \text{dress}, \text{pink}) \rightarrow \textit{Knows}(x, y)$.

[7] 8. A triangular number (or triangle number) counts the objects that can form an equilateral triangle, as in the diagram below. The n^{th} triangular number is the sum of the first n integers, as shown in the following figure illustrating the first four triangular numbers (what is the fifth one?):



Design a circuit that takes a 4-bit unsigned binary integer $x_3x_2x_1x_0$ as input, and outputs True if this integer is a triangular number, and False otherwise. You may assume that 0 is not a triangular number.