CPSC 121 Midterm 2
Tuesday November 12th, 2013

Name: ____________________ Student ID: ____________________

Signature: ____________________

– You have 70 minutes to write the 6 questions on this examination.
  A total of 50 marks are available.

– **Justify all of your answers.**

– You are allowed to bring in one hand-written, double-sided 8.5 x 11in sheet of notes, and nothing else.

– Keep your answers short. If you run out of space for a question, you have written too much.

– The number in square brackets to the left of the question number indicates the number of marks allocated for that question. Use these to help you determine how much time you should spend on each question.

– Use the back of the pages for your rough work.

– **Good luck!**

Question	Marks
1	
2	
3	
4	
5	
6	
Total	

UNIVERSITY REGULATIONS:

– Each candidate should be prepared to produce, upon request, his/her UBC card.

– No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

– CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

  1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
  2. Speaking or communicating with other candidates.
  3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

– Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

[9] 1. Short Answers

[2] a. What kind of proof is the following (note: part of the proof is missing and has been replaced by ...) ?

> Consider an unspecified positive integer a. Suppose that the cube root of a is a rational number, but not an integer. This means that ... And therefore the cube root of a is irrational.

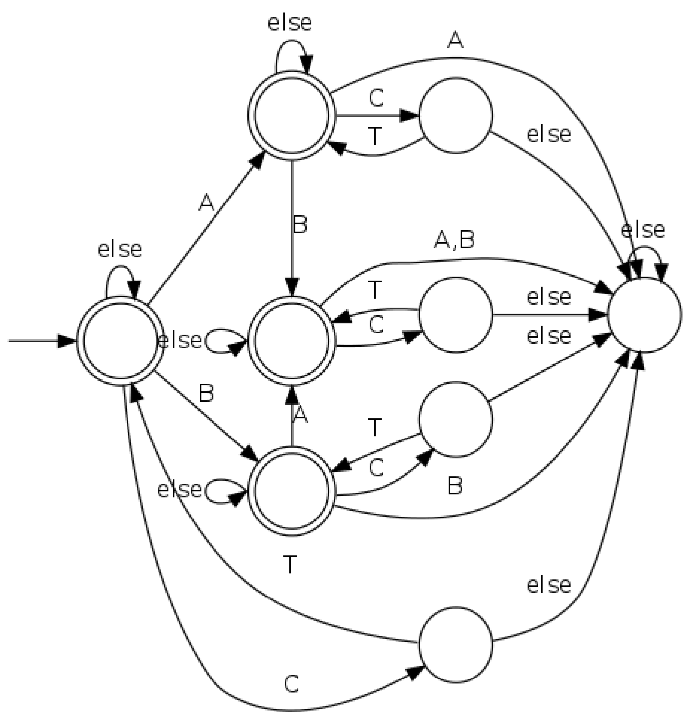
[3] b. Consider the following statement:

$$\forall x \in \mathbf{Z}, \exists y \in \mathbf{Z}, Q(x, y)$$

Can a proof that starts with "Choose y = 3." be valid? Explain why or why not.

[4] c. You are asked to write a direct proof to show that the statement $\exists x \in D, \exists y \in D, P(x, y)$ is **false**. What would your proof look like? Write as much of the proof as you can, given that we did not tell you what $P(x, y)$ is.

[7] 2. Consider the following deterministic finite-state automaton, whose inputs are words written using uppercase letters (A through Z) only. The word "else" on an edge denotes every letter that does not appear on any other edge leaving from the same node as that edge (for instance, the word "else" on the edge that joins the leftmost node to itself is the same as "D, E, F, ..., Y, Z").



[4] a. Which of the following words will this finite-state automaton accept (circle one of Yes/No for each word)?

- **ACTIVE** Yes No
- **DICTATION** Yes No
- **DICTATORIAL** Yes No
- **FOUGHT** Yes No
- **LUMBER** Yes No
- **PEBBLE** Yes No
- **RABIES** Yes No
- **SCHNITZEL** Yes No

[3] b. Describe as simply as you can the set of words that this finite-state automaton accepts.

[12] 3. For each of the following inferences, first state if it's valid or invalid, and then justify your answer briefly. Assume that a is an element of domain D.

[3] a.

$$\begin{array}{l} \exists x \in D, P(x) \rightarrow Q(x) \\ \sim Q(a) \\ \hline \therefore \sim P(a) \end{array}$$

[3] b.

$$\begin{array}{l} \forall x \in D, P(x) \rightarrow Q(x) \\ \sim Q(a) \\ \hline \therefore \sim P(a) \end{array}$$

[3] c.

$$\begin{array}{l} \exists y \in D(\exists x \in D, P(x,y)) \rightarrow Q(y) \\ \forall z \in D, P(z,a) \\ \hline \therefore\ Q(a) \end{array}$$

[3] d.

$$\begin{array}{l} \forall x \in D, \sim \forall y \in D, P(x,y) \rightarrow Q(x,y) \\ \hline \therefore\ \forall x \in D, \exists y \in D, P(x,y) \wedge \sim Q(x,y) \end{array}$$

[10] 4. For each of the two following theorems, first translate the theorem into predicate logic, and then prove it using a proof technique of your choice.

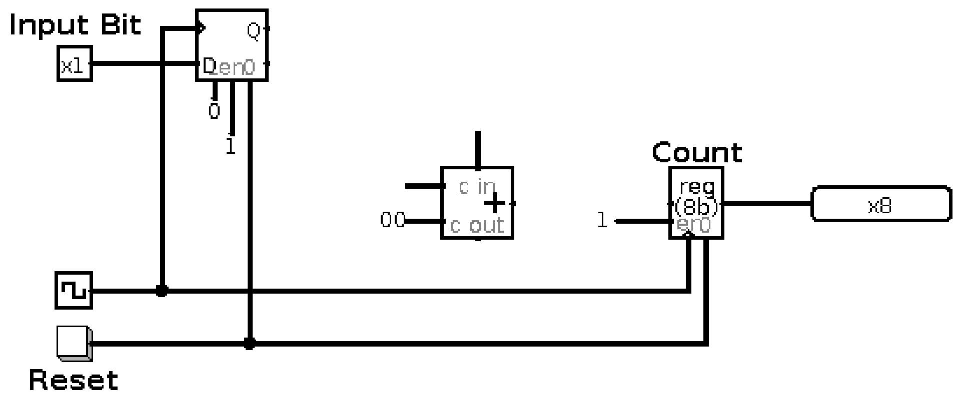
a. It is possible to find a pair of integers whose sum and difference are both perfect squares.

b. If a, b and c are positive integers, and ac divides bc, then a divides b.

[6] 5. One of your teaching assistants designed a circuit to count the number of distinct sequences of consecutive 1 bits in a multi-bit input (the next bit of the input is available every time the clock ticks). For instance, the output of the circuit should be

- 1 if the input was 11100,
- 2 if the input was 0111101,
- 3 if the input was 10010010,

etc. Unfortunately a crazy Computer Science instructor armed with an eraser came by! Help the teaching assistant by restoring the gate(s) and/or wire(s) the instructor has deleted, to get a working circuit.



[6] 6. Using a proof by contradiction, prove that for all positive integers a and b, $(a+b)/2 \geq \sqrt{ab}$. Hints: (1) you should square both sides of an inequality, (2) what is $(a-b)^2$?