[0] 1. Consider the logical equivalence rule $p \wedge q \equiv \sim(\sim p \vee \sim q)$. For each of the following, circle **APPLIES** to indicate this rule **can** be applied **directly** to the given statement or **CANNOT APPLY** to indicate it cannot. If the rule can be applied, provide the resulting statement.

1. $\overset{p}{(d \vee c)} \wedge \overset{q}{\sim r}$ **CANNOT APPLY** **APPLIES** with result: $\sim(\sim(d \vee c) \vee \sim(\sim r))$

2. $w @ v$ **CANNOT APPLY** **APPLIES** with result:

3. $\sim(\sim \overset{p}{m} \vee \sim \overset{q}{n})$ **CANNOT APPLY** **APPLIES** with result: $m \wedge n$

4. $a \to (\overset{p}{b} \wedge \overset{q}{c})$ **CANNOT APPLY** **APPLIES** with result: $a \to (\sim(\sim b \vee \sim c))$

[4] 2. Each statement below is a "contingency": true for some assignments of truth values to the variables and false for others. For each statement, give an assignment of truth values that makes the statement true and another that makes it false.

	Statement	True assignment	False assignment
1.	$\sim a \rightarrow a$	$a =$ T	$a =$ F
2.	$(p \wedge q) \rightarrow \sim(q \rightarrow (r \vee p))$	$p =$ F, $q =$ T, $r =$ T	$p =$ T, $q =$ T, $r =$ T
3.	$(p \vee (\sim q)) \oplus (q \wedge (r \rightarrow s))$	$p =$ T, $q =$ F, $r =$ T, $s =$ T	$p =$ T, $q =$ T, $r =$ F, $s =$ T

F T

T $\oplus$ T

F

[4] 3. Complete the following table where each row shows a single integer value represented as a "normal" decimal number, a 6-bit unsigned binary number, a 6-bit signed binary number, and a 2-digit unsigned hexadecimal number.

Fill in each blank, unshaded entry. **Write N/A if a value cannot be represented in some column.**

Decimal	6-bit unsigned	6-bit signed	2-digit unsigned hexadecimal
-15	N/A	110001	
87	N/A	N/A	57
250	N/A		FA
-21		101011	N/A
161	N/A		A1
31	011111		1F
	010110	010110	16

[0] 4. Consider the following definitions describing a world of time travelers who may observe events in different orders:

- $I \equiv \{e_1, e_2, e_3, \ldots\}$ is the set of "events" (important things that have happened)
- $P \equiv \{\text{Adric}, \text{Ben}, \text{Clara}, \text{Doc}, \ldots\}$ is the set of people
- $\text{ObsOrder}(p, i, j)$ means person p observes that event i occurs before event j
- $\text{Before}(i, j)$ means event i actually happened before event j
- $\text{At}(p, i)$ means person p was at event i
- $\text{Met}(p, q)$ means person p met person q

State the following in predicate logic:

(a) Ben has not met Adric, and Adric has not met Ben.

$\sim \text{Met}(\text{Ben}, \text{Adric}) \wedge \sim \text{Met}(\text{Adric}, \text{Ben})$

(b) Clara was at every event.

$\forall i \in I, \text{At}(\text{Clara}, i)$

(c) Doc has observed two different events occurring in both possible orders.

$\exists i \in I, \exists j \in I, i \neq j \wedge \text{ObsOrder}(\text{Doc}, i, j) \wedge \text{ObsOrder}(\text{Doc}, j, i)$

(d) Everyone who was at an event observed that event to occur before some other event (but the other event is not necessarily the same for everyone).

$\forall i \in P, \forall p \in P, \text{At}(p, i) \rightarrow (\exists j \in I, \text{ObsOrder}(p, i, j) \wedge i \neq j)$

Assuming—for just this part—that the statements above are true, evaluate the truth of the following statements. (Circle one of **TRUE**, **FALSE**, or **UNKNOWN**.)

(a) **TRUE** **FALSE** **UNKNOWN** $\text{Before}(e_7, e_1)$

(b) **TRUE** **FALSE** **UNKNOWN** $\text{Met}(\text{Ben}, \text{Adric}) \leftrightarrow \text{Met}(\text{Adric}, \text{Ben})$

(c) **TRUE** **FALSE** **UNKNOWN** $\text{At}(\cancel{\phi}\ \text{Clara}, e_3)$

(d) **TRUE** **FALSE** **UNKNOWN** $\text{ObsOrder}(\cancel{\phi}\ \text{Doc}, e_2, e_1)$

Now, define the following predicates using ObsOrder, Before, At, and Met:

(a) Known(p_1, p_2) means that p_1 and p_2 have met each other (both ways).
Known(p_1, p_2) $\equiv$ $Met(p_1, p_2) \land Met(p_2, p_1)$

(b) Nexus(i) means that everyone was at event i.
Nexus(i) $\equiv$ $\forall p \in P, At(p, i)$

(c) Traveler(p) means that person p observed some distinct pair of events to occur in the opposite order to the way they actually happened. *Note: "distinct pair" means "two things that are different from each other".*
Traveler(p) $\equiv$ $\exists i \in I, \exists j \in I, i \neq j \land ObsOrder(p, i, j) \land Before(j, i)$

(d) Companion(p) means that person p is a Traveler who was at more than one event with Doc (but is not Doc).
Companion(p) $\equiv$ $Traveler(p) \land p \neq Doc \land$
$\exists i \in I, \exists j \in I, i \neq j \land At(p, i) \land At(p, j) \land At(Doc, i) \land At(Doc, j)$.

[0] 5. Consider the following theorem with a missing premise:

1. $p \wedge (s \to r)$
2. $\sim(r \wedge q)$
3. $q \leftrightarrow \sim d$
4. **MISSING**

$\therefore \sim s$

$p \to q$
$\sim q$
$\therefore \sim p$

$\sim r \vee \sim q$

$q \leftrightarrow \sim d \equiv (q \to \sim d) \wedge (\sim d \to q)$

We know only that the missing premise mentions the variable d in some way but does not mention p.

Because of the missing premise, we cannot prove the theorem. **Instead, give at least two *substantially different*, promising approaches to starting the proof.** Each approach should show at least one step beyond a premise or before the conclusion, and you should briefly justify why each step is promising.

(a) **Approach:** Use MT on $s \to r$ and $\sim r$ (a new goal) to derive $\sim s$.

Brief justification: Working backward from conclusion.

(b) **Approach:** Use SPEC on premise #1 to derive $s \to r$

Brief justification: Two reasons:

1. Need $s \to r$ for approach #1
2. Do <u>not</u> need p, since p is only mentioned in one premise.