THE UNIVERSITY OF BRITISH COLUMBIA

CPSC 121: MIDTERM 1 – Group – February 12, 2013

Important notes about this examination

1. **You have 40 minutes to complete this exam.**
2. No electronic aides (e.g., phones or calculators) are allowed in the exam.
3. Each student may bring up to three 8.5x11" sheets of paper with any information of the student's choice on them, typed or printed, one side or both, although the sheets must be readable without magnification!
4. Please see the regulations regarding student conduct during examinations on the opposite side of this page!
5. All students in your designated group collaborate to produce a single submission.
6. Good luck!

Full Name:________________________

Signature:________________________

UBC Student #:

Full Name:________________________

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UBC Student #:

Full Name:________________________

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Please do not write in this space:

Question 1:

Question 2:

Question 3:

Question 4:

Question 5:

Question 6:

Question 7:

Question 8:

Question 9:

Question 10:

Student Conduct during Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. No questions will be answered in this exam. If you see text you feel is ambiguous, make a reasonable assumption, write it down, and proceed to answer the question.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
   i. speaking or communicating with other examination candidates, unless otherwise authorized;
   ii. purposely exposing written papers to the view of other examination candidates or imaging devices;
   iii. purposely viewing the written papers of other examination candidates;
   iv. using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
   v. using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)—(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

[4] 1. Circle the letter of the statement that best translates prompts (i)–(vi) using the definitions:

- $g \equiv$ I can say George's last name correctly.
- $s \equiv$ I can say Steve's last name without laughing.
- $h \equiv$ I love CPSC 121. (We chose h for "heart".)

(CPSC 121 instructor George's last name is hard to pronounce while Steve's is funny.)

i. I can't say George's last name correctly.

(a) $\sim g$
(b) g
(c) s
(d) $\sim s$

ii. I love CPSC 121 but I cannot say Steve's last name without laughing.

(a) $h \oplus \sim s$
(b) $h \wedge \sim s$
(c) $h \vee \sim s$
(d) $h \rightarrow \sim s$

iii. $\sim(s \wedge h)$

(a) I can't say Steve's last name without laughing, and I love CPSC 121.
(b) It's not true that I can say Steve's name without laughing and I love CPSC 121.
(c) I can say Steve's last name without laughing, or I love CPSC 121.
(d) I can't say Steve's last name without laughing, and I don't love CPSC 121.

iv. $g \oplus s$

(a) I know how to pronounce George's last name, but I cannot say Steve's last name without laughing.
(b) I can say exactly one of the instructors' names without any problems.
(c) I know how to pronounce George's last name, or I can say Steve's last name without laughing, or both.
(d) I know how to pronounce George's last name but not Steve's.

[4] 2. Each statement below is a "contingency": true for some assignments of truth values to the variables and false for others. For each statement, give an assignment of truth values that makes the statement true and another that makes it false.

	Statement	**True** assignment	**False** assignment
1.	$\sim a$	$a =$	$a =$
2.	$\sim(p \vee \sim q)$	$p =\quad, q =$	$p =\quad, q =$
3.	$p \leftrightarrow (q \wedge (\sim p \vee \sim r))$	$p =\quad, q =\quad, r =$	$p =\quad, q =\quad, r =$

[3] 3. Draw a direct translation of the following propositional logic expression into a circuit; label the output `out`.

$$(a \wedge \sim b) \vee (a \wedge b)$$

[3] 4. Consider the **rule of inference**:

$$\begin{array}{l} p \rightarrow q \\ \underline{\sim q} \\ \therefore \sim p \end{array}$$

For each of the following, circle **APPLIES** to indicate the rule **can** be applied **directly** to the given statements or **CANNOT APPLY** to indicate it cannot. If the rule can be applied, provide the resulting statement.

NOTE: each problem starts with **two** statements to which we may be ably to the rule.

1. $\sim(a \wedge b) \rightarrow c, \sim c$ **CANNOT APPLY** **APPLIES** with result:

2. $a \rightarrow (b \vee c), \sim c$ **CANNOT APPLY** **APPLIES** with result:

3. $g \vee (h \rightarrow i), \sim i$ **CANNOT APPLY** **APPLIES** with result:

[6] 5. Complete the following table where each row shows a single integer value represented as a "normal" decimal number, a 6-bit unsigned binary number, a 6-bit signed binary number, and a 2-digit unsigned hexadecimal number.

Fill in each blank, unshaded entry. **Write N/A if a value cannot be represented** in some column.

Decimal	**6-bit unsigned**	**6-bit signed**	**2-digit unsigned hexadecimal**
11			
-1			
		111010	
		000111	
			25
	100000		

[11] 6. Consider the following definitions describing a world of time travelers who may observe events in different orders:

- $I \equiv \{\mathrm{e}_1, \mathrm{e}_2, \mathrm{e}_3, \ldots\}$ is the set of "events" (important things that have happened)
- $P \equiv \{\mathrm{Adric}, \mathrm{Ben}, \mathrm{Clara}, \mathrm{Doc}, \ldots\}$ is the set of people
- $\mathrm{ObsOrder}(p, i, j)$ means person p observes that event i occurs before event j
- $\mathrm{Before}(i, j)$ means event i actually happened before event j
- $\mathrm{At}(p, i)$ means person p was at event i
- $\mathrm{Met}(p, q)$ means person p met person q

State the following in predicate logic:

(a) Doc has met himself.

(b) No event actually happened before itself.

Assuming—for just this part—that the statements above are true, evaluate the truth of the following statements. (Circle one of **TRUE**, **FALSE**, or **UNKNOWN**.)

(a) **TRUE** **FALSE** **UNKNOWN** $\mathrm{Before}(\mathrm{e}_1, \mathrm{e}_1)$

(b) **TRUE** **FALSE** **UNKNOWN** $\mathrm{Met}(\mathrm{Doc}, \mathrm{Clara})$

(c) **TRUE** **FALSE** **UNKNOWN** $\mathrm{At}(\mathrm{Doc}, \mathrm{e}_1) \rightarrow \mathrm{Met}(\mathrm{Doc}, \mathrm{Doc})$

Now, define the following predicates using ObsOrder, Before, At, and Met:

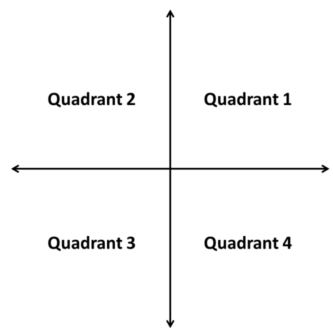
(a) $\text{Streamed}(i, j)$ means that event i and j actually happened in some order: one or the other first (but not both or neither).
$\text{Streamed}(i, j) \equiv$

(b) $\text{Next}(i, j)$ means that event i actually happened before event j and no event actually happened between events i and j.
$\text{Next}(i, j) \equiv$

[11] 7. A Cartesian coordinate is an x-value and a y-value like $(3, -2)$, which is 3 units in the positive direction (**right**, $\rightarrow$) on the x-axis and 2 units in the negative direction (**up**, $\uparrow$) on the y-axis.

In the 16-bit "**VariableSplit**" representation, the first 3 bits are an unsigned binary number $a = a_1a_2a_3$ representing "the number of extra bits in the x-value". The 13 remaining bits $n_1n_2n_3 \ldots n_{13}$ are divided into two signed binary integers with lengths dependent on a. First the x-value, which is $3 + a$ bits long: $n_1n_2n_3 \ldots n_{3+a}$, and then the y-value, which is $10 - a$ bits long: $n_{4+a}n_{5+a} \ldots n_{13}$.

We name the "quadrants" of the xy-plane with the following chart, where anything on the x-axis or y-axis counts as "quadrant 0":



(a) What is the largest (positive) x-value representable with **VariableSplit**? (Write your answer in base 10.)

(b) Give two distinct **VariableSplit** bit patterns that both represent the origin $(0, 0)$.

(c) **Using a 1-of-8 multiplexer** sketch a circuit that takes as input bits n_4, n_5, n_6, n_7, n_8, n_9, n_{10}, n_{11}, and a_1, a_2, and a_3 you need from a **VariableSplit** value and produces the value of the first bit in the y-value (labeled n_{4+a} above). *Hint: a correct circuit is not very complex!*

(d) **Design** a circuit (i.e., give a propositional logic formula for each bit of the output $q = q_1 q_2 q_3$ but do not draw the circuit diagram) that takes as input the first bit of the x-value x_1 and the first bit of the y-value y_1 from a **VariableSplit** value and produces the quadrant that coordinate is in, **assuming it is not in quadrant 0**. Your circuit's output should be an unsigned binary number 1–4.

Note: your circuit gets x_1 and y_1 as input; it doesn't need to figure out what they are!

[7] 8. Complete the following propositional logic proof to prove u:

1. $q \wedge m$	premise
2. $\sim r \vee \sim(m \wedge \sim u)$	premise
3. $q \rightarrow (r \wedge s)$	premise

YOU MIGHT WANT TO PUT SCRATCH WORK DOWN HERE.

[6] 9. Consider the following propositional logic statement:

$((p \wedge q) \rightarrow (\sim q \wedge r)) \wedge p$

Simplify this statement, and **prove with a logical equivalence proof** that the original statement is equivalent to your simplified form.

Hint: the statement's simplest form has only two variables, each occuring only once.

PROOF:

$((p \wedge q) \rightarrow (\sim q \wedge r)) \wedge p \equiv$

YOU MIGHT WANT TO PUT SCRATCH WORK DOWN HERE.

[5] 10. **Two extra-fun BONUS questions just for groups!** (These are worth bonus marks... but finish the rest of the exam first; it's worth **WAY** more points-per-unit-effort than the bonus.)

Refer to the definitions in the problem about time-travelers and write the statement "everyone who was at the beginning of time (the event that actually happened first) met each other". For bonus credit the statement must be written both correctly and concisely and must not suggest that anyone met themselves.

Refer to the documentation on our 16-bit **VariableSplit** Cartesian coordinate representation and give the coordinates of the point furthest from the origin that is representable as a **VariableSplit** number. Briefly justify your answer.

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If you write answers here (or anywhere other than their intended location), mark them clearly, indicate which question they respond to, and indicate at the provided solution blank for that question where you wrote your solution.

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