CPSC 121 Quiz 3
Wednesday, 2012 July 18

Name: ______________________ Student ID: ______________________
Signature: ______________________
Your signature acknowledges your understanding of and agreement to the rules below.

- You have 40 minutes to write the 3 questions on this examination. A total of 20 marks are available.
- You may have as an aide up to 3 textbooks and a 3 inch stack of paper notes and nothing else. **No electronic devices allowed**; so, no cell phones and no calculators.
- Keep your answers short. If you run out of space for a question, you have likely written too much.
- The number in square brackets to the left of the question number indicates the number of marks allocated for that question. Use these to help you plan your use of time on the exam.
- Clearly indicate your answer to each problem. If your answer is not in the provided blank, then indicate where the answer is, and at the answer's location indicate the question it addresses.
- **Good luck!**

Question	Marks
1	
2	
3	
Total	

UNIVERSITY REGULATIONS:

- Each candidate should be prepared to produce, upon request, his/her UBC card.
- No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.
- CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
  2. Speaking or communicating with other candidates.
  3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

[6] 1. Given the premises: $(\sim q \to \sim p) \wedge (q \to r)$, $\sim q \wedge r$, $d \to (s \wedge p)$, and $s \vee d$; prove s.

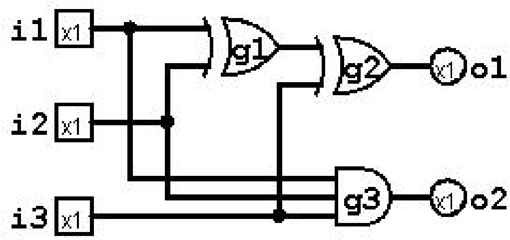
You may only use rules listed on "Dave's Awesome Sheet" (or lemmas you prove yourself on this quiz). Please make your finished proof clear and easy to read. **We suggest using the back of the previous page for your scratchwork!** We have begun the proof below.

Proof (that the conclusion reached below follows from the premises listed):

1.	$(\sim q \to \sim p) \wedge (q \to r)$	[premise]
2.	$\sim q \wedge r$	[premise]
3.	$d \to (s \wedge p)$	[premise]
4.	$s \vee d$	[premise]

[6] 2. We can model the connections in combinational circuits with predicate logic. Let P be the set of "ports"—inputs of gates, outputs of gates, inputs of the circuit, and outputs of the circuit. Let $\mathit{WireTo}(a, b)$ defined for $a \in P, b \in P$ mean that a has a wire leading to b. (Notice that WireTo has a *direction*. For example, we'd never have a "WireTo" relationship *from* any port *to* an input of the circuit!) Let $\mathit{Input}(a)$ and $\mathit{Output}(a)$—each defined for $a \in P$—mean that a is an input to or output from the circuit, respectively.

Consider this circuit (with gates labeled g_1, g_2, and g_3):



In this circuit, for example, $\mathit{Input}(i_1)$ is true because i_1 is an input to the circuit. However, g_1 (a gate) and o_1 (an output of the circuit) are not inputs; so, $\mathit{Input}(g_1)$ and $\mathit{Input}(o_1)$ are both false.

Specifically: the Input predicate is true for i_1, i_2, and i_3 and false otherwise. The Output predicate is true for o_1 and o_2 and false otherwise. Here are the only true WireTo relationships: $\mathit{WireTo}(i_1, g_1)$, $\mathit{WireTo}(i_1, g_3)$, $\mathit{WireTo}(i_2, g_1)$, $\mathit{WireTo}(i_2, g_3)$, $\mathit{WireTo}(i_3, g_2)$, $\mathit{WireTo}(i_3, g_3)$, $\mathit{WireTo}(g_1, g_2)$, $\mathit{WireTo}(g_2, o_1)$, and $\mathit{WireTo}(g_3, o_2)$.

[1] (a) $\mathit{Input}(g_2) \rightarrow \mathit{WireTo}(i_1, g_2)$ is (circle one): **TRUE** **FALSE**
Remember how conditionals work!

[2] (b) **Dis**prove the statement $\forall a \in P$, $\mathit{WireTo}(a, o_1)$.

[3] (c) **Prove** the statement $\forall a \in P, \sim \mathit{Output}(a) \rightarrow \exists b \in P$, $\mathit{WireTo}(a, b)$.

[8] 3. A "binary tree" is composed of "subtrees". Each subtree is either a leaf (with no children), or it has a left child and a right child, each of which is itself a subtree.

Let S be the set of all subtrees in a particular binary tree.
Let $\mathit{LeftChild}(p, c)$ defined for $p \in S, c \in S$ be true exactly when c is the *left* child of p.
Let $\mathit{RightChild}(p, c)$ defined for $p \in S, c \in S$ be true exactly when c is the *right* child of p

Now, let's further define what it means to be a binary tree.

You may always use the predicates $\mathit{LeftChild}$ and $\mathit{RightChild}$. **Furthermore, in each part, you may rely on the definitions and statements from the previous parts being correct, regardless of whether you completed them correctly!**

[2] (a) Define a predicate $\mathit{Child}(p, c)$ (defined for $p \in S, c \in S$) that is true exactly when c is a child of p.

$\mathit{Child}(p, c) =$

[2] (b) Define a predicate $\mathit{Root}(s)$ (defined for $s \in S$) that is true when s has no parent.

$\mathit{Root}(s) =$

[2] (c) No subtree has more than two children. State that fact in predicate logic.

[2] (d) Every non-root subtree in a binary tree has a parent. State that fact in predicate logic.

BONUS: Earn up to 2 bonus points by doing one or more of these problems.

- A different way to think of a binary tree—closer to the typical Java version—is that it is composed (mostly) of *nodes* rather than *subtrees*. There's also one special value `null` that is not a node (or anything else) at all. So, for example, in this version every node has two children, but a node may have `null` as one or both children (and be a leaf, in the latter case). Consisder the statement "every tree has a root"; discuss whether it is true or false in the two "versions" of binary trees and then rewrite the binary tree problem with this version, explaining the differences.
- Part (c) of the circuit problem is a general statement that we could apply to any combinational circuit. Must it necessarily be true—or necessarily be false—for all combinational circuits? **Should** it be true or false for all combinational circuits? Justify your answers, ideally including an illustrative example or two.