CPSC 121 :: 2011S :: Midterm 1 :: 2011.07.06

NAME: SOLUTION, STATS & NOTES
SIGNATURE:
STUDENT NUMBER:

- There are 11 pages in total.
- You have 60 minutes.
- A total of 50 (+2) marks are available.
- As a suggestion, allocate ≈1 minute per mark.
- You may want to complete what you consider to be the easiest questions first!
- No notes or electronic devices are permitted.
- Use the backs of pages if you require additional space, and clearly identify when you have done so.

UNIVERSITY REGULATIONS:

- Each candidate should be prepared to produce, upon request, his/her university-issued ID.
- No candidate shall be permitted to enter the examination room after the expiration of one half hour or to leave during the first half hour of the examination.
- CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
  2. Speaking or communicating with other candidates.
  3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	TOTAL

Question 1. [5 marks]
average: **3.8** (median: 4, mode: 5)

part a. [1 mark]
Convert 111010101101000010_2 to HEX

SOLUTION
$3AB42_{16}$
Dave's Note:
Group them into blocks of 4 ... starting on the right: $11\ \ 1010\ \ 1011\ \ 0100\ \ 0010 = 3\ \ A\ \ B\ \ 4\ \ 2$

part b. [2 marks]
Take the last 4 digits of your student number (as a base 10 number) and convert it to binary.

part c. [2 marks]
Take the negation of the first 4 digits of your student number (as a base 10 number), and convert it to a 16-bit signed (2's complement) binary number.

Question 2. [10 marks]
average: **4.2** (median: 4, mode: 4)

part a. [2 marks]
While drinking at Moe's Tavern, Professor Frink admitted that he generated his bank card 4-digit PIN randomly using only the digits 1 and 0. Knowing that a card is invalidated after 7 incorrect guesses, if you obtained his bank card, what is the probability you could access his account? Give it as a rational (fraction) number in regular base 10.

SOLUTION
There are 4 bits, so 16 possible PINs, you get 7 guesses, so $7/16$.

part b. [2 marks]
Represent your answer in (a) as a floating point number in base 2. (for example, $\frac{1}{2} = 0.1$):

SOLUTION
0.0111

part c. [2 marks]
Take your 4-digit bank card PIN and convert it to binary *(just kidding!)*.
What's the minimum number of bits you need to be able to store a 4-digit PIN?

SOLUTION
14 bits
Dave's Note:
With k bits, you can represent the numbers $0...2^k - 1$, or in other words, to represent the numbers $0...n$, you need $k = \log_2 n$ bits. In this case, you can can just look up on the table for the first power of two greater than 9999.

part d. [2 marks]
A bank has added the 6 buttons A-F to their ATMs and now allows all HEX digits in their PINs...
What's the minimum number of bits they need to store their PINs?

SOLUTION
16 bits
Dave's Note:
Each HEX digit corresponds to 4 bits.

part e. [2 marks]
The bank from (d) also wants to identify uninitialized bank cards (no PIN yet) using a binary representation. What would you recommend they do? (be brief)
SOLUTION
They need to add at least 1 more bit per PIN (17 bits in total), and (for example) use that bit as a special bit to identify uninitialized PINs.

Question 3. [7 marks]
average: **4.3** (median: 5, mode: 5)

Wednesday night is logic night at Moe's Tavern. One night Lenny and Carl were arguing over which of their rules of inference were correct:

Lenny's Rule:	Carl's Rule:
$\dfrac{p \oplus q}{\therefore\ p \vee q}$	$\dfrac{p \oplus q}{\therefore \sim p \vee \sim q}$

Homer, convinced that both Lenny and Carl got it all backwards, came up with his own rule. Barney, thinking Homer is brilliant, generalized Homer's reasoning:

Homer's Rule:	Barney's Rule:
$\begin{array}{c} p \vee q \\ \sim p \vee \sim q \\ \hline \therefore\ p \oplus q \end{array}$	$\begin{array}{c} a \to b \\ a \to c \\ \hline \therefore\ (c \wedge b) \to a \end{array}$

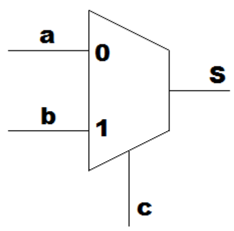
Can you identify whose rules are valid?
Note: (as is the policy at Moe's Tavern) you only have to justify why someone's rule is invalid.

SOLUTION
Lenny, Carl and Homer's rules are all valid. Barney's rule is invalid, as can be shown in the case where $a \equiv F, b \equiv c \equiv T$: The premises are both true but the conclusion is false.

Dave's Note:
If you can't see by inspection, you can prove each of the valid rules by using a truth table.
For example, for Lenny's rule, show:
$(p \oplus q) \to (p \vee q)$
is a tautology (every row is true), and for Homer's rule, show:
$((p \vee q) \wedge (\sim p \vee \sim q)) \to (p \oplus q)$
is a tautology. For Barney's rule, show:
$((a \to b) \wedge (a \to c)) \to ((c \wedge b) \to a)$
is not a tautology (it has a false row, as shown above).

Question 4. [8 marks]
average: **3.4** (median: 2, mode: 2)

You are currently a manager at a company that designs circuits using only 2-input NAND gates. You have been approached by a traveling gate salesman that is selling 3-input MUX (multiplexer / select) gates that have similar characteristics to your NAND gates (speed, power consumption, etc.). Her gate is shown below. She says that because her MUX gates have 3 inputs they are better than your 2 input NAND gates. She also *claims* that because the formula for the MUX gate $[s \equiv (a \wedge \sim c) \vee (b \wedge c)]$ has a NOT, AND and OR in it, the MUX is universal just like your current NAND gates. She says that if you sign an exclusivity agreement to *only* use her MUX gates, she can offer them to you at a fraction (one tenth) of the cost of your NAND gates. Do you accept her offer? Why or why not? (justify your answer)



SOLUTION

Yes, I'd buy her MUX gates – equally fast and efficient, and yet quite a bargain! Because the MUX is universal they can be used in place of our other universal gate (the NAND).

We should verify her claim that that the MUX is universal:
To obtain $(\sim p)$, set $a = T, b = F, c = p$. $\therefore$ $s \equiv \sim p$.
To obtain $(p \wedge q)$, set $a = F, b = p, c = q$. $\therefore$ $s \equiv (p \wedge q)$.
And to obtain $(p \vee q)$, set $a = p, b = T, c = q$.

Dave's Note:

- The strongest argument is that with 2 MUXs we can simulate a NAND gate, and replace all of our previous gates for $1/5$ the cost.
- One could argue that this *direct* substitution would make our circuits twice as slow, but don't forget that it takes 2 NANDs to make an AND and 3 NANDS to make an OR, where each can be made with a single MUX, so we're still probably much better off with the MUXs.
- Arguments against taking the deal are:
  - 3-input gates could increase the wire complexity of the circuits (unlikely)
  - The increased cost in redesigning existing circuits and re-training existing staff.
  - If you have a strong aversion to exclusivity agreements.

Question 5. [8 marks]
average: **4.1** (median: 4, mode: 4)

Prove the absorption law using other logical equivalence Laws:
$p \wedge (p \vee q) \equiv p$

SOLUTION

RHS	$\equiv p$	
	$\equiv p \vee F$	I
	$\equiv p \vee (q \wedge \sim q)$	NEG
	$\equiv (p \vee q) \wedge (p \vee \sim q)$	DIST
	$\equiv (p \vee q) \wedge (p \vee \sim q) \wedge (p \vee q)$	ID
	$\equiv (p \vee (q \wedge \sim q)) \wedge (p \vee q)$	DIST
	$\equiv (p \vee (F)) \wedge (p \vee q)$	NEG
	$\equiv p \wedge (p \vee q)$	I
	$\equiv$ LHS	

Dave's Note:
My tip to tackle this question... you need to create a $(p \vee q)$ from thin air... so generate one, make a copy, then remove the first.

An even shorter solution:

LHS	$\equiv p \wedge (p \vee q)$	
	$\equiv (p \vee F) \wedge (p \vee q)$	I
	$\equiv p \vee (F \wedge q)$	DIST
	$\equiv p \vee F$	UB
	$\equiv p$	I
	$\equiv$ RHS	

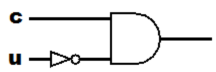
Question 6. [6 marks]
average: **4.4** (median: 4.5, mode: 6)

Given the following propositional statements:

c : You cheated on this midterm
d : Dave is a great teacher
u : You are a great student

and the following two premises:

1. **Either** you are a great student **or** you cheated on this midterm.
2. The output of the following circuit is false:



re-write the premises as boolean formulas, and then prove using the rules of inference and equivalence laws that given the above premises, the following conclusion is valid:

if Dave is a great teacher **then** you are a great student.

SOLUTION

1.	$u \oplus c$	premise
2.	$\sim(c \wedge \sim u)$	premise
3.	$\therefore\ (c \wedge \sim u) \vee (\sim c \wedge u)$	1 + definition of $\oplus$
4.	$\therefore\ (\sim c \wedge u)$	ELIM 2 + 3
5.	$\therefore\ u$	SPEC 4
6.	$\therefore\ d \rightarrow u$	GEN$\rightarrow$

Dave's Note:

It was my intent that the first premise be an XOR, but that was confusing to some... here is the solution with inclusive or:

1.	$u \vee c$	premise
2.	$\sim(c \wedge \sim u)$	premise
3.	$\therefore \sim c \vee \sim\sim u$	DM 2
4.	$\therefore \sim c \vee u$	DNEG
5.	$\therefore\ u \vee u$	RES 1 + 4
6.	$\therefore\ u$	ID 5
7.	$\therefore\ d \rightarrow u$	GEN$\rightarrow$

Question 7. [2 marks]
average: **0.4** (median: 0, mode: 0)

You are using the magic box in the lab, and need one more NOT gate to complete your circuit. Unfortunately, someone has spilled Coke Zero™ over all of your NOT chips and, while they are now delicious, they no longer work. The only chips you have remaining are XOR chips. Can you use an XOR chip to simulate a NOT gate? (justify your answer)

SOLUTION
Yes I can use an XOR chip. We can force one of the inputs to be true.

the output of an XOR is $(p\wedge \sim q) \vee (\sim p \wedge q)$. If we set $q = T$, then the output is:
$(p\wedge \sim T) \vee (\sim p \wedge T)$
$\equiv \sim p$

Question 8. [4 marks]
average: **2.3** (median: 3, mode: 3)

In assignment 2, you generated the truth table and corresponding circuit for a half-adder. Generate a truth table and circuit for a half-subtractor. There will be two inputs (x_0, y_0) and two outputs (d, b), where d is the difference $(x_0 - y_0)$ and b is a *borrow* bit similar in concept to the carry bit in adders.

SOLUTION

x_0	y_0	d	b
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Question 9. [Bonus 2 marks]

part a. [Bonus 1 mark]
How would you like your remaining discretionary class participation marks determined: (circle one)

1. Tutorial attendance
2. The discretion of the lab TAs
3. WebCT/Vista participation
4. Increase the % of my assignment marks
5. Increase the % of my midterm marks
6. Increase the % of my quiz marks

part b. [Bonus 1 mark]
Please respond as honestly as you can: (circle one per row)

	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree	
This exam was too long	1	2	3	4	5	average: **4.0**
This exam was too hard	1	2	3	4	5	average: **4.1**
The exam content was fair	1	2	3	4	5	average: **3.3**

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