CPSC 121 Midterm 2
Tuesday, March 15th, 2011

Name: ______________________ Student ID: ______________________
Signature: ______________________ Section (circle one): Patrice Steve

- You have 75 minutes to write the 7 questions on this examination. A total of 50 marks are available.
- **Justify all of your answers.**
- You are allowed to bring in one hand-written, double-sided 8.5 x 11in sheet of notes, and nothing else.
- Keep your answers short. If you run out of space for a question, you have written too much.
- The number in square brackets to the left of the question number indicates the number of marks allocated for that question. Use these to help you determine how much time you should spend on each question.
- Use the back of the pages for your rough work.
- **Good luck!**

Question	Marks
1	
2	
3	
4	
5	
6	
7	
Total	

UNIVERSITY REGULATIONS:

- Each candidate should be prepared to produce, upon request, his/her library card.
- No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.
- CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
  2. Speaking or communicating with other candidates.
  3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

[6] 1. For the first statement in each of the following pairs of statements, suggest a plausible proof approach (that is, a plan involving one or more of the proof techniques you have learned, such as proof of existence [witness proof], generalizing from the generic particular [WLOG], or direct proof [antecedent assumption]) that makes as much progress as possible on the proof. Note that you **do not** have enough information to actually prove the theorem!

For the second, slightly altered statement, explain why the statement will probably be easy to prove or disprove.

[3] a. Suggest an approach for: $\forall x \in D, P(x) \rightarrow Q(x)$

Explain why this will be easy to prove/disprove: $\exists x \in D, P(x) \rightarrow Q(x)$

[3] b. Suggest an approach for: $\exists x \in D, \exists y \in D, (x \neq y \wedge R(x, y))$

Explain why this will be easy to prove/disprove: $\forall x \in D, \forall y \in D, (x \neq y \wedge R(x, y))$

[7] 2. Prove that for every distinct pair of real numbers, there is another real number that is between them (greater than the smaller one and less than the larger one). Hint: a direct proof works well.

[6] 3. Each senior instructor in the department of Computer Science either always lies, or always tells the truth. Both Steve and Patrice are senior instructors in the department.

- Steve says: “Exactly one of us is lying.”
- Patrice says: “Steve is telling the truth.”

Using a proof by contradiction, show that Steve is lying.

[8] 4. Consider the following proposition:

$$\sim\forall x \in S, (\exists y \in C, F(x, y)) \rightarrow P(x)$$

[3] a. Use the following definitions to translate the given proposition into English:

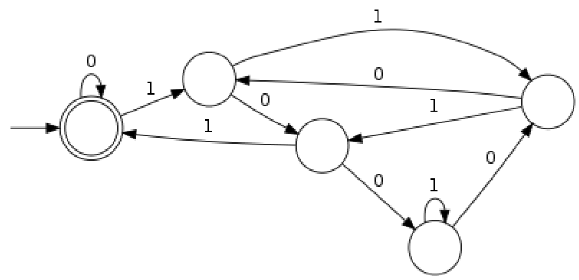
- S is the set of all students in the Faculty of Science at UBC
- C is the set of all UBC courses
- $F(x, y)$ means that student x failed course y
- $P(x)$ means that x will be put on academic probation

(For full marks, your translation must be reasonably natural-sounding, rather than a word for word translation.)

[5] b. Prove that the given proposition is logically equivalent to

$$\exists x \in S, (\exists y \in C, F(x, y)) \wedge \sim P(x)$$

[7] 5. Consider the following DFA:



[5] a. Which of the following strings of bits will be accepted by the DFA (write Yes or No next to each string)?

[1] i. 1001

[1] ii. 1010

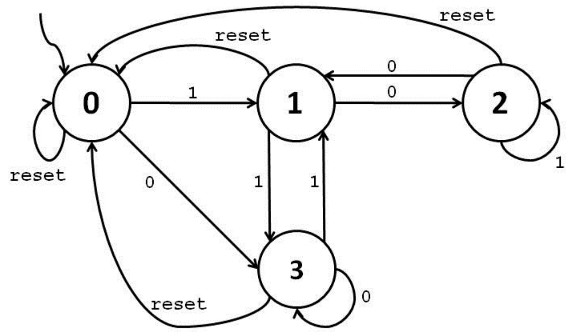
[1] iii. 1111

[1] iv. 10101

[1] v. 11001

[2] b. Using the examples from part (a) as a guide, explain what sequences of bits will be accepted by the DFA. Hint: think of them as unsigned integers.

[8] 6. Consider the following DFA:

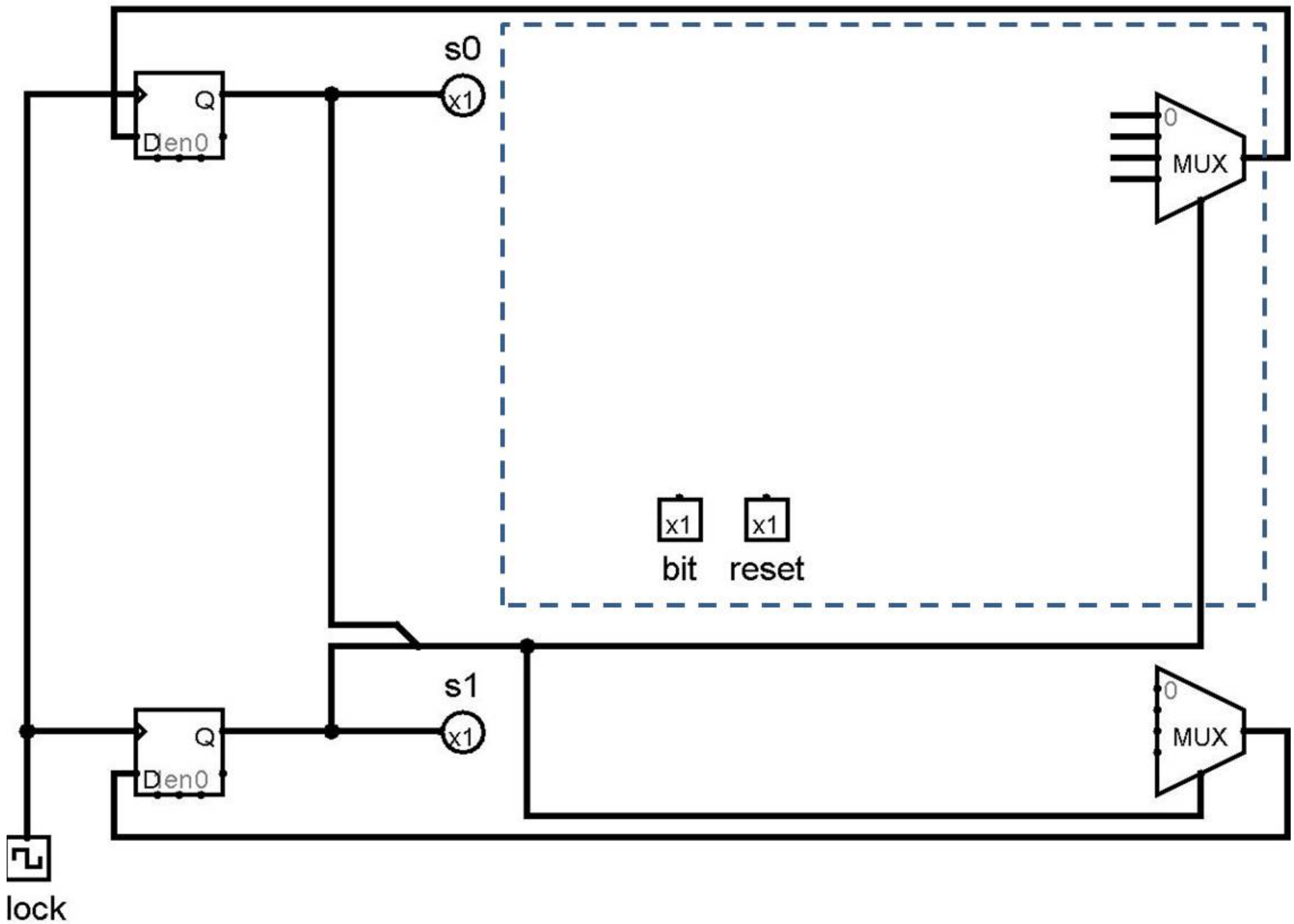


If the reset column in the table below is 1 then the input to the DFA is `reset` (the bit column is ignored). Otherwise, the input is the value in the bit column. We would like to implement a circuit corresponding to this DFA.

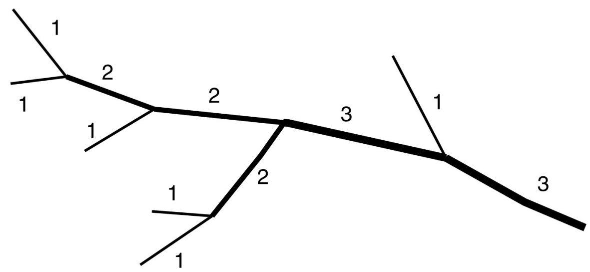
[2] a. Fill in the incomplete rows of the truth table below describing this DFA's behaviour. (The state number is included to clarify the meaning of "s0" and "s1".)

State	s0	s1	bit	reset	new s0	new s1
0	0	0	0	0		
0	0	0	0	1		
0	0	0	1	0		
0	0	0	1	1		
1	0	1	0	0		
1	0	1	0	1		
1	0	1	1	0		
1	0	1	1	1		
2	1	0	0	0	0	1
2	1	0	0	1	0	0
2	1	0	1	0	1	0
2	1	0	1	1	0	0
3	1	1	0	0	1	1
3	1	1	0	1	0	0
3	1	1	1	0	0	1
3	1	1	1	1	0	0

[6] b. Implement the portion of the sequential circuit below necessary to calculate the next value for the flip-flop storing s0 (i.e., the portion enclosed in the dashed rectangle).



[8] 7. This question considers a classification system for rivers. Here's a river marked up with this system:



We call the numbers SNs. To use SNs, we divide a river into segments. Each segment flows either from a starting point of the river or from where two or more other segments join. Each segment ends either where it flows into the ocean or where it joins with one or more other segments.

The SN of a segment (which is a positive integer) comes from these rules:

(a) If the segment flows from a starting point, then its SN is 1.

(b) If it flows from where two or more river segments join, then:

  i. if one of the joining segments has a SN of i and the others have smaller SNs, then the resulting segment's SN is i.

  ii. if two or more of the joining segments' SNs are i and no others have larger SNs, then the resulting segment's SN is $i + 1$.

Note: you may assume that the river system of a segment that flows from where other segments join is larger than the river systems of the segments that join together.

Problem: Prove by induction that any river segment with SN n flows from at least 2^{n-1} different starting points. (Even an incorrect proof with appropriate form will receive substantial partial credit.)

Name	Rule(s)
Identity Laws	$p \wedge T \equiv p$ $p \vee F \equiv p$
Domination Laws	$p \wedge F \equiv F$ $p \vee T \equiv T$
Idempotent Laws	$p \wedge p \equiv p$ $p \vee p \equiv p$
Commutative Laws	$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$
Associative Laws	$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$ $p \vee (q \vee r) \equiv (p \vee q) \vee r$
Distributive Laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Absorption Laws	$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$
Negation Laws	$p \wedge \sim p \equiv F$ $p \vee \sim p \equiv T$
Double Negation Law	$\sim (\sim p) \equiv p$
De Morgan's Laws	$\sim (p \wedge q) \equiv (\sim p) \vee (\sim q)$ $\sim (p \vee q) \equiv (\sim p) \wedge (\sim q)$

Powers of 2	
n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	62
7	128
8	256
9	512
10	1024
11	2048
12	4096
13	8192
14	16384
15	32768
16	65536