CPSC 121 Midterm 1
Thursday, February 3rd, 2011

Name: ______________________ Student ID: ______________________
Signature: ______________________ Section (circle one): Patrice Steve

– You have 75 minutes to write the 10 questions on this examination. A total of 65 marks are available.

– **Justify all of your answers.**

– You are allowed to bring in one hand-written, double-sided 8.5 x 11in sheet of notes, and nothing else.

– **Logical equivalences, rules of inference and powers of 2 are on the last page.**

– Keep your answers short. If you run out of space for a question, you have written too much.

– The number in square brackets to the left of the question number indicates the number of marks allocated for that question. Use these to help you determine how much time you should spend on each question.

– Use the back of the pages for your rough work.

– **Good luck!**

Question	Marks
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

UNIVERSITY REGULATIONS:

– Each candidate should be prepared to produce, upon request, his/her UBC card.

– No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

– CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
2. Speaking or communicating with other candidates.
3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

– Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

[1] 1. Do you want your tutorial attendance to count toward your grade? If you answer 'yes', then 1% of your final course mark will be based on your tutorial attendance (# attended divided by total number minus 2, but no more than full credit). Online quizzes would be worth only 5% rather than 6%. If you answer 'no', then tutorial attendance is worth nothing in your mark, and online quizzes remain a full 6% of your final course mark.

[6] 2. Translate the following statements from English into propositional logic. Give an appropriate definition for any variables you use. No variable is allowed to correspond to a proposition that contains logical operators (for instance, for part (a), you can not use "r: Either I wear my wedding ring on my left hand or I wear it on my right hand"). You may circle portions of the printed statement and write the variable name beside the circle to reduce writing time.

[2] a. Either I wear my wedding ring on my left hand or I wear it on my right hand

[2] b. Naomi did not like the movie, but she really liked the popcorn.

[2] c. If a voting system is Pareto efficient and exhibits Independence of Irrelevant Alternatives, then it must be a dictatorship.

[6] 3. Translate the following statements from propositional logic back into English. Use the following definitions:

w:	Werewolves are better than ghosts.	g:	Ghosts are better than werewolves.
s:	Steve is a werewolf.	p:	Patrice is a ghost.
r:	Werewolves are real.		

[2] a. $w \oplus g$

[2] b. $s \rightarrow (r \wedge w)$

[2] c. $p \leftrightarrow g$

[7] 4. Determine what the following four premises:

$$w \oplus g$$
$$s \rightarrow (r \wedge w)$$
$$p \leftrightarrow g$$
$$w$$

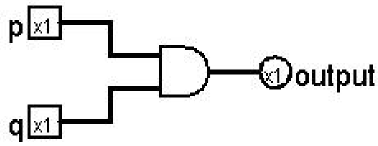
indicate about the truth of p (whether Patrice is a ghost!). That is, prove p, prove $\sim p$, or show that both are consistent with the premises. You may skip steps involving only the commutative, associative, and double negation laws, but you should show all other steps. You need not write the names of the rules you use.

Note: you may use any of the following logical equivalences for exclusive OR and the biconditional, but no others. If you would like to refer to them, use the names E1 to E4.

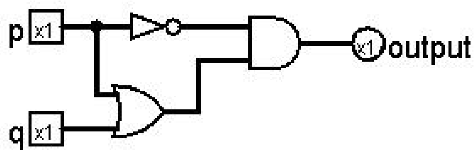
E1 : $x \oplus y \equiv (x \wedge \sim y) \vee (\sim x \wedge y)$ **E3** : $x \leftrightarrow y \equiv (x \rightarrow y) \wedge (y \rightarrow x)$
E2 : $x \oplus y \equiv (x \vee y) \wedge \sim(x \wedge y)$ **E4** : $x \leftrightarrow y \equiv (x \wedge y) \vee (\sim x \wedge \sim y)$

[9] 5. Translate the following three circuits into propositional logic statements. (Do not simplify the propositional logic statements; leave them in the same form as the circuits.)

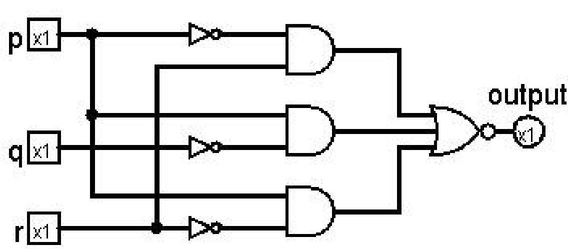
[2] a.



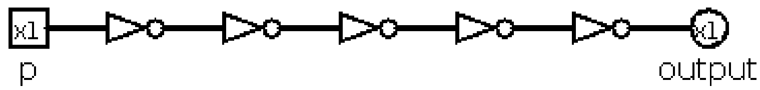
[3] b.



[4] c.



[6] 6. Consider the following circuit:



[1] a. Write the logical expression that corresponds to this circuit. (Do not simplify the propositional logic statement; leave it in the same form as the circuit.)

[1] b. Now simplify this logical expression as much as possible.

[1] c. Draw the circuit corresponding to the simpler expression.

[3] d. What does our model of combinational circuits (i.e., propositional logic) fail to account for that causes these two circuits to behave differently?

[6] 7. Consider the following predicates:

V(x,y): person x voted for person y during the last elections.
E(x,y): person x likes to eat food y.

where the domain for people and foods are denoted by P and F respectively. Translate each of the following quantified statements into English:

[2] a. $\forall x \in P, V(x, \text{“Stephen Harper”}) \rightarrow E(x, \text{“Carrots”})$

[2] b. $\exists x \in F, \forall y \in P, V(y, \text{“Michael Ignatieff”}) \rightarrow \sim E(y, x)$

[2] c. $\forall x \in P, \exists y \in F, E(x, y) \wedge \sim E(\text{“Jack Layton”}, y)$

[6] 8. Prove that $\sim q \vee \sim (p \vee r) \equiv \sim (p \wedge q) \wedge (q \rightarrow \sim r)$ using logical equivalences (NOT truth tables). You may skip steps involving only the commutative, associative, and double negation laws, but you should show all other steps. You need not write the names of the rules you use.

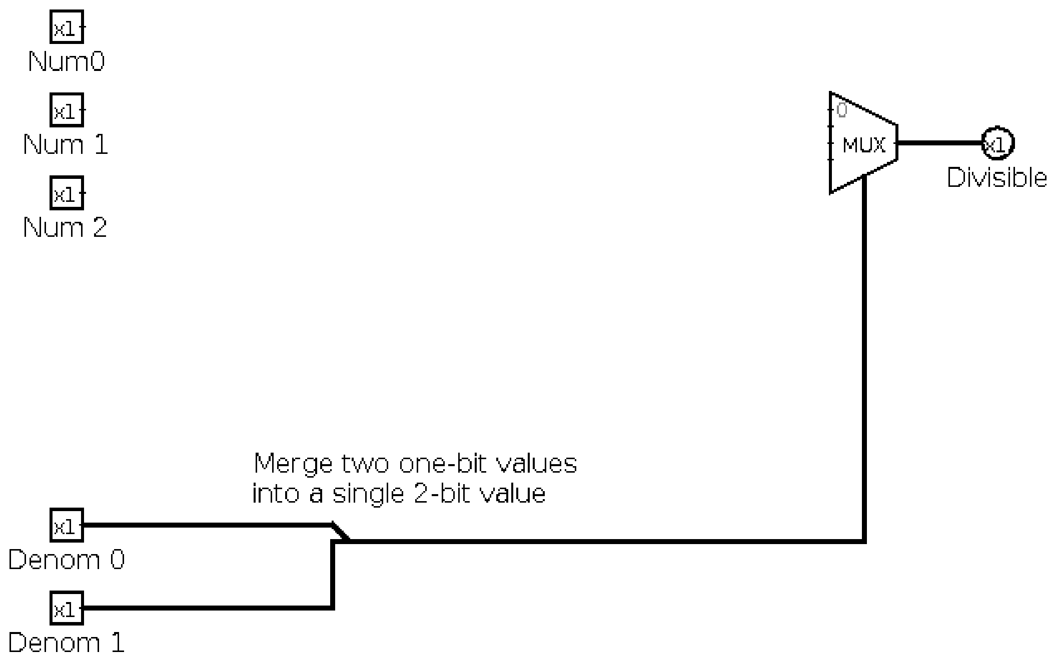
[12] 9. This question considers the problem of determining whether an unsigned binary integer x is divisible by another unsigned binary integer y (recall that x is divisible by y if x/y is an integer). Assume for this question that no integer is divisible by 0, and that 0 is divisible by every other integer.

[4] a. Build a circuit that takes a 3-bit unsigned binary number x and determines whether it is divisible by 2. You may use inverters (NOT gates), and AND, OR, NAND, and NOR gates with any number of inputs.

[4] b. Build a circuit that takes a 3-bit unsigned binary number x and determines whether it is divisible by 3. You may use inverters (NOT gates), and AND, OR, NAND, and NOR gates with any number of inputs.

[4] c. Complete the following circuit that takes a 3-bit unsigned binary number x and a 2-bit unsigned binary number y and determines whether x is divisible by y. Assume the "divisible by 2" and "divisible by 3" subcircuits work (correctly) as described in the previous two problems. If you use one of these, represent it by a rectangle containing the label `db2` or `db3`.

(Reminder: assume for this question that no integer is divisible by 0, and that 0 is divisible by every other integer.)



[6] 10. The human ear can detect sounds that range from frequencies of about 20 Hz to about 20000 Hz. However, it takes larger absolute differences in frequency toward the higher frequency values for us to differentiate between sounds. For example, the difference between 40 Hz and 80 Hz is MUCH clearer than the difference between 10000 Hz and 10040 Hz, even though both are 40 Hz apart.

A friend proposes to represent the frequency of a sound (which is intended to be used for playing that sound to a human) as a 12-bit signed binary number.

Explain at least two different important problems with this representation choice.

1.

2.

Name	Rule(s)
Identity Laws	$p \wedge T \equiv p$ $p \vee F \equiv p$
Domination Laws	$p \wedge F \equiv F$ $p \vee T \equiv T$
Idempotent Laws	$p \wedge p \equiv p$ $p \vee p \equiv p$
Commutative Laws	$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$
Associative Laws	$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$ $p \vee (q \vee r) \equiv (p \vee q) \vee r$
Distributive Laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Absorption Laws	$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$
Negation Laws	$p \wedge \sim p \equiv F$ $p \vee \sim p \equiv T$
Double Negation Law	$\sim (\sim p) \equiv p$
De Morgan's Laws	$\sim (p \wedge q) \equiv (\sim p) \vee (\sim q)$ $\sim (p \vee q) \equiv (\sim p) \wedge (\sim q)$

Powers of 2	
n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	62
7	128
8	256
9	512
10	1024
11	2048
12	4096
13	8192
14	16384
15	32768
16	65536

Rule of Inference	Name
$p \wedge q$ $\therefore p$	Simplification
p $p \to q$ $\therefore q$	Modus ponens
$\sim q$ $p \to q$ $\therefore \sim p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	Hypothetical syllogism
$p \vee q$ $\sim p$ $\therefore q$	Disjunctive syllogism
$p \to r$ $q \to r$ $\therefore (p \vee q) \to r$	Rule for proof by cases.