CPSC 121: Models of Computation
Midterm Exam #1, 2009 October 8

Name: ______________________ Student ID: ______________________

Signature: ______________________

- You have **75 minutes** to write the 7 questions on this exam.
- A total of **60 marks** are available. You may want to complete what you consider to be the easiest questions first!
- Ensure that you clearly indicate a legible answer for each question.
- You are allowed any reasonable number of textbooks and quantity of notes as references. Otherwise, no notes, aides, or electronic equipment are allowed.
- Good luck!

UNIVERSITY REGULATIONS

1. Each candidate must be prepared to produce, upon request, a UBCcard for identification.
2. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
3. No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
4. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:
   - having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;
   - speaking or communicating with other candidates; and
   - purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
5. Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
6. Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Total
6	4	8	10	12	10	10	60

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If you put solutions here or anywhere other than the blank provided for each solution, you must *clearly* indicate which problem the solution goes with and also indicate where the solution is at the designated area for that problem's solution.

1 Representation Schemes [6 marks]

We wish to represent propositional logic statements in binary. Our representation uses 4-bit values to represent four symbols—$\sim$, $\wedge$, $\vee$, (, and)—and numbered variables like p_0, p_1, p_2, and so forth. Symbols are represented according to the following table:

Symbol	Representation
$\sim$	1000
$\wedge$	1010
$\vee$	1011
(	1100
)	1101

Variables are represented by 4-bit patterns that start with a 0, where a particular variable p_i is represented by the unsigned binary representation of the number i. (So, for example, $p_3 = 0011$.)

For example, the logical expression $\sim(p_0 \wedge p_1)$ would translate to 100011000000101000011101 as follows:

$\sim$	(	p_0	$\wedge$	p_1	)
1000	1100	0000	1010	0001	1101

1. Circle the best reason to reserve bit patterns starting with 1 only for symbols. **[2 marks]**

   (a) The representation scheme couldn't have been done any other way.

   (b) This scheme makes it easy to tell symbols from variables.

   (c) This scheme maximizes the number of variable numbers represented, given the length of the patterns and the number of symbols represented.

   (d) All of these are good reasons.

2. How many different variables can be represented in this scheme? **[1 mark]**

3. We represent variable numbers as unsigned numbers. Circle the best reason *not* to have the variable numbers be signed? (That is, the last three bits of any 4-bit pattern that starts with a 0 would be the 3-bit signed binary representation of the variable's number.) **[1 mark]**

   (a) The four-bit patterns for variables start with 0; so, we can't use signed numbers.

   (b) Signed numbers are generally more complex to work with than unsigned.

   (c) We would not be able to represent as many distinct variables using signed numbers.

   (d) All of these are good reasons.

4. The hexadecimal values 6 and C each represent a single variable or symbol in our encoding scheme. What are they? **[2 marks]**

2 Exploring Consequences of Propositional Logic Statements [4 marks]

Consider the following premises:

$q \wedge u$
$\sim(r \vee \sim p)$
$q \vee s$
$s \rightarrow (r \wedge p)$
$u \leftrightarrow (x \vee w)$
$x \rightarrow w$

For each of the following expressions, indicate whether—given the premises above—we know that the expression is true, know that the expression is false, know the expression is both true and false, or don't know the value of the expression. **[1 mark per question]**

1. u

   Value: TRUE FALSE BOTH UNKNOWN

2. x

   Value: TRUE FALSE BOTH UNKNOWN

3. $x \vee p$

   Value: TRUE FALSE BOTH UNKNOWN

4. $x \wedge \sim w$

   Value: TRUE FALSE BOTH UNKNOWN

3 Circuit Design [8 marks]

Sketch a circuit that takes a 3-bit unsigned binary number as input and outputs true if and only if the input is a prime (2, 3, 5, or 7). Use only inverters and two-input AND, OR, NAND, NOR, XOR, and XNOR gates in your solution. (Note: Show your work to receive partial credit. . . and help you avoid mistakes!)

4 Circuits, Correctness, and Equivalency [10 marks]

This problem focuses on a circuit to multiply two 2-bit unsigned numbers. The result is a 3-bit unsigned number plus an overflow flag. If the product of the inputs fits in 3 bits, the 3-bit output is the product and the overflow flag is false. Otherwise, the 3-bit output's value doesn't matter and the overflow flag is true.

The following truth table correctly specifies the circuit's behaviour, *but is not necessarily the only correct specification!* In the truth table, we multiply $x * y$ to produce z. For clarity, the truth table includes columns for x, y, and z's values as decimal numbers as well as binary numbers.

x	x_1	x_0	y	y_1	y_0	z	z_2	z_1	z_0	$overflow$
0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	1	0	0	0	0	0
0	0	0	2	1	0	0	0	0	0	0
0	0	0	3	1	1	0	0	0	0	0
1	0	1	0	0	0	0	0	0	0	0
1	0	1	1	0	1	1	0	0	1	0
1	0	1	2	1	0	2	0	1	0	0
1	0	1	3	1	1	3	0	1	1	0
2	1	0	0	0	0	0	0	0	0	0
2	1	0	1	0	1	2	0	1	0	0
2	1	0	2	1	0	4	1	0	0	0
2	1	0	3	1	1	6	1	1	0	0
3	1	1	0	0	0	0	0	0	0	0
3	1	1	1	0	1	3	0	1	1	0
3	1	1	2	1	0	6	1	1	0	0
3	1	1	3	1	1	9*	0	0	0	1

(The last entry in z's column is 9 because $3 * 3 = 9$; however, the last $z_2z_1z_0$ entry is not equal to 9.)

Answer the following questions about this problem:

1. Which of the following propositional logic statements correctly describes the output z_2? **Circle all correct answers. [4 marks]**

   (a) $(x_1 \wedge \sim x_0 \wedge y_1 \wedge \sim y_0) \vee (x_1 \wedge \sim x_0 \wedge y_1 \wedge y_0) \vee (x_1 \wedge x_0 \wedge y_1 \wedge \sim y_0)$

   (b) $(x_1 \oplus x_0) \wedge y_1$

   (c) $x_1 \wedge y_1$

   (d) $\sim(x_1 \wedge x_0 \wedge y_1 \wedge y_0)$

2. Can two correct implementations of the overflow output be logically equivalent? Circle the best answer. **[2 marks]**

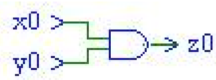
   (a) Yes, they must be.

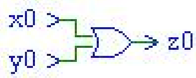
   (b) They can be but aren't necessarily.

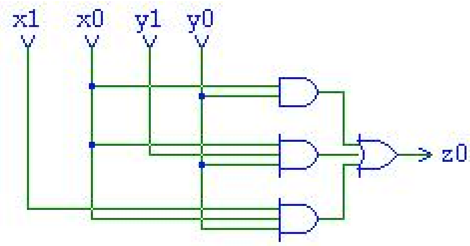
   (c) No, they cannot be.

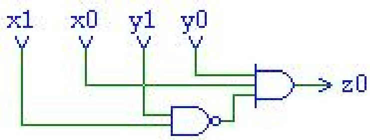
   (d) There's not enough information to tell.

3. Which of the following circuits correctly implements the output z_0? **Circle all correct answers. [4 marks]**

(a) 

(b) 

(c) 

(d) 

5 Propositional Logic Proof [12 marks]

1. **Without proceeding with the proof**, propose and justify two promising approaches to proving $p \wedge w$ (i.e., two alternative starting points in planning your proof) given the following premises. **[4 marks]**

1.	$r \vee s$	
2.	$\sim s \rightarrow \sim x$	
3.	$(\sim y \wedge r) \rightarrow p$	
4.	$\sim s$	Approach #1:
5.	$\sim y \vee s$	
6.	$w \vee x$	
7.	$y \rightarrow q$	

Approach #2:

2. Complete the following formal propositional logic proof that $\sim p \wedge q$ follows from the premises listed below. Use only logical equivalence laws and rules of inference in Epp Chapter 1 plus our "definition of conditional" and "definition of biconditional". You need not explicitly show steps that rely only on the double negation, commutative, and associative logical equivalence rules. **[6 marks]**

1.	$\sim(u \vee \sim q \vee s)$	premise
2.	$\sim r \leftrightarrow (\sim s \vee p)$	premise
3.	$a \rightarrow (b \vee c)$	premise
4.	$p \rightarrow u$	premise
5.	$q \vee r$	premise
6.	$(\sim r \rightarrow (\sim s \vee p)) \wedge ((\sim s \vee p) \rightarrow \sim r)$	Def'n of bicond on 2

3. Consider the following possibly flawed proof that r follows from the two premises shown. Circle each line of the proof that *does not* follow from the indicated previous lines *or* indicate that no line is flawed. **[2 marks]**

1.	$\sim(p \wedge q)$	premise
2.	$(\sim p \wedge \sim q) \rightarrow r$	premise
3.	$\sim p$	specialization on 1
4.	$\sim p \wedge (\sim p \vee \sim q)$	absorption on 3
5.	$\sim p \vee \sim q$	specialization on 4
6.	r	modus ponens on 5 and 2

6 Grab Bag [10 marks]

1. Is the conclusion of a propositional logic proof **logically equivalent** to the conjunction of its premises? **[2 marks]**

   (a) Yes, it must be logically equivalent, and the proof formally establishes that it is.

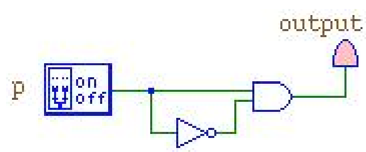
   (b) Yes, it must be logically equivalent, but the proof does not establish that it is.

   (c) It may or may not be logically equivalent, and the proof does not establish whether it is.

   (d) No, it is not logically equivalent, but the proof does not formally establish that it's not.

   (e) No, it is not logically equivalent, and the proof formally establishes that it is not.

2. Consider the following circuit: **[3 marks]**



   According to our propositional logic model, the output of this circuit should always be false. Briefly explain what unmodeled behavior of real circuits makes it so the output will not always be false. (For full credit, indicate under exactly what circumstances this circuit's output becomes true.)

3. Which of the following are equivalent to $p \leftrightarrow \sim q$? **Circle all correct answers. [3 marks]**

   (a) $(p \to q) \wedge (\sim p \to \sim q)$
   (b) $(p \to \sim q) \wedge (\sim q \to p)$
   (c) $p \oplus q$
   (d) $(p \wedge q) \vee (\sim p \wedge \sim q)$
   (e) $(\sim p \vee q) \wedge (p \vee \sim q)$
   (f) $(\sim p \wedge q) \vee (p \wedge \sim q)$

4. Let D be the domain of 8-bit signed ints, *not* mathematical integers. (Bear in mind what you've learned about how arithmetic on signed binary numbers works, and where it differs from arithmetic on actual integers!)

   Is the following statement true? **[2 marks]**

   $\forall x \in D, \forall y \in D, (x > y) \to (x - y > 0)$.

   (a) Definitely true, which we can see by subtracting y from both sides of $x > y$.
   (b) Definitely true, as illustrated by $x = 1$ and $y = 127$.
   (c) Definitely true, but none of the reasons here prove it.
   (d) Definitely false, because if y is a large positive number, the subtraction may "overflow".
   (e) Definitely false, as illustrated by $x = 126$ and $y = -10$.
   (f) Definitely false, but none of the reasons here prove it.
   (g) Not enough information to tell.

   Powers of two reminder: In case they're handy for this question, here are several powers of two:

2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}
1	2	4	8	16	32	64	128	256	512	1024

7 Predicate Logic [10 marks]

Consider the following predicates, defined as appropriate over the sets of all student IDs: S; the set of all courses: C; and the set of all programs of study P:

$$
\begin{aligned}
HasCredit(s,c) &\equiv \text{The student with ID } s \text{ has earned passing credit for course } c. \\
Required(p,c) &\equiv \text{Students in program } p \text{ must earn passing credit for course } c \text{ to complete the program.} \\
HasRequirements(s,p) &\equiv \text{The student with ID } s \text{ has earned passing credit for all of program } p\text{'s required courses.}
\end{aligned}
$$

1. Which of these correctly defines $HasRequirements$ in terms of $HasCredit$ and $Required$. **[2 marks]**

   (a) $HasRequirements(s,p) \equiv \exists c \in C, Required(p,c) \rightarrow HasCredit(s,c)$

   (b) $HasRequirements(s,p) \equiv \exists c \in C, Required(p,c) \wedge HasCredit(s,c)$

   (c) $HasRequirements(s,p) \equiv \forall c \in C, Required(p,c) \rightarrow HasCredit(s,c)$

   (d) $HasRequirements(s,p) \equiv \forall c \in C, Required(p,c) \wedge HasCredit(s,c)$

   (e) None of these

2. According to the following definition, what does it mean for a course to be a "possible requirement"? (A literal translation is worth partial credit; for full credit succinctly explain the meaning in English.) **[2 marks]**

   $PossibleRequirement(p,c) \equiv \sim Required(p,c) \;\wedge\; \forall s \in S, HasRequirements(s,p) \rightarrow HasCredit(s,c)$

3. Imagine the set of all courses were CPSC 121, MATH 200, and BSKT 444. Alice has earned passing credit for CPSC 121 and MATH 200 but not BSKT 444. Bob has earned credit for CPSC 121 and BSKT 444 but not MATH 200. There are other students, but we don't know which courses they've taken. For each of the following statements, indicate whether it's known to be true, known to be false, or not known to be either true or false. Briefly justify your answer. **[2 marks per question]**

   (a) $\forall c \in C, \exists s \in S, HasCredit(s,c)$

   Value: TRUE FALSE UNKNOWN

   Justify:

   (b) $\exists s \in S, \forall c \in C, HasCredit(s,c)$

   Value: TRUE FALSE UNKNOWN

   Justify:

   (c) $\forall s \in S, \forall c \in C, HasCredit(s,c)$

   Value: TRUE FALSE UNKNOWN

   Justify:

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