CPSC 121 Sample Midterm Examination
October 2007

Name: _____ Student ID: _____
Signature: _____

– You have 65 minutes to write the 7 questions on this examination. A total of 60 marks are available.

– **Justify all of your answers.**

– You are allowed to bring in one hand-written, double-sided 8.5 x 11 sheet of notes, and nothing else.

– Keep your answers short. If you run out of space for a question, you have written too much.

– The number in square brackets to the left of the question number indicates the number of marks allocated for that question. Use these to help you determine how much time you should spend on each question.

– Use the back of the pages for your rough work.

– **Good luck!**

Question	Marks
1	
2	
3	
4	
5	
6	
7	
Total	

UNIVERSITY REGULATIONS:

– Each candidate should be prepared to produce, upon request, his/her library card.

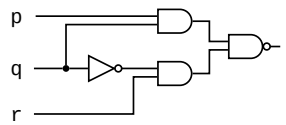
– No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

– CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
2. Speaking or communicating with other candidates.
3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

– Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

[9] 1. Match each proposition in the left column with the logically equivalent proposition in the right column. Each proposition from the left column is equivalent to only one proposition in the right column. Since there are three propositions in the left column and five propositions in the right column, two of the propsitions in the right column will not be used when you make your matches. Justify your answers (i.e. show why the two propositions are logically equivalent).

a.	$(p \vee q) \rightarrow r$
b.	$(p \oplus q) \vee (p \wedge q) \vee r$
c.	

1.	T
2.	$(p \vee q) \vee (\sim q \wedge r)$
3.	$p \vee (q \vee r)$
4.	$(\sim q \wedge \sim p) \vee r$
5.	$p \oplus q \oplus r$

[10] 2. Rules of inference

[3] a. Prove that the following rule (simplification) is a valid rule of inference:

$p \wedge q$

$\therefore p$

[7] b. Prove or disprove that the following argument is valid. Please clearly state the rule of inference you are using for each step.

1. $m \rightarrow n$
2. $k \rightarrow m$
3. $r \rightarrow q$
4. $g \wedge h$
5. $\sim r \rightarrow m$
6. $h \rightarrow \sim (k \vee q)$

$\therefore n$

[6] 3. Translate each of the following English propositions into predicate logic. Assume that C is the set of all countries, M is the set of all continents, $B(x, y)$ means x shares a border with y, $P(x, y)$ means country x is in continent y, and $H(x)$ means country x is in the Northern Hemisphere.

[3] a. No country in the Northern Hemisphere borders a country in the Southern Hemisphere.

[3] b. There are some countries in the Northern Hemisphere that share a border but aren't in the same continent.

[11] 4. Representing numbers.

[2] (a.) Give the binary (bit-string) representation of the following base 10 numbers when represented as four-bit, signed numbers in two's complement notation.

(i.) 5:

(ii.) -5:

[2] (b.) Here is a truth table (with numbers 1 and 0 instead of T and F) for a circuit that adds two two-bit, **unsigned** numbers (i.e., $a_1a_0 + b_1b_0 = u_1u_0$). (It is split in two parts to fit on the paper better.)

a_1	a_0	b_1	b_0	u_1	u_0	s_1	s_0
0	0	0	0	0	0		
0	0	0	1	0	1		
0	0	1	0	1	0		
0	0	1	1	1	1		
0	1	0	0	0	1		
0	1	0	1	1	0		
0	1	1	0	1	1		
0	1	1	1	0	0		

a_1	a_0	b_1	b_0	u_1	u_0	s_1	s_0
1	0	0	0	1	0		
1	0	0	1	1	1		
1	0	1	0	0	0		
1	0	1	1	0	1		
1	1	0	0	1	1		
1	1	0	1	0	0		
1	1	1	0	0	1		
1	1	1	1	1	0		

What changes are required to this circuit for it to add two **signed, two's complement** numbers (ie., s_1s_0)? To answer this question you can fill in the s_1 and s_0 columns in the table or provide a careful explanation of what values these column should have, whichever you find easier. In either case, explain your answer carefully.

[7] (c.) Draw a circuit that takes two two-bit, signed, two's complement numbers a_1a_0 and b_1b_0 as input and computes an output of 1 if $a < b$ and 0 otherwise.

[5] 5. Answer the following questions about these four sets.

$$\begin{aligned} A &= \{1, 2, ..., 10\} \\ B &= \{x \in A \mid x \text{ is odd}\} \\ C &= \{x \in A \mid x \text{ is divisible by } 3\} \\ D &= \{1, 2, 1, 3, 3, 3\} \end{aligned}$$

(a.) $A \cap B =$

(b.) $B \cup C =$

(c.) $|B \cup D| =$

(d.) $|D - B| =$

(e.) $|\{T \mid T \subseteq C\}| =$

[11] 6. Consider the following statement:

> An unsigned integer represented in binary is odd if and only if its rightmost (least-significant) bit is 1.

[3] (a) Rewrite the statement using predicate logic (quantifiers and predicates).

[8] (b) Prove the truth of the statement using a direct proof. Part of the marks will be given for the structure of the proof; the rest will be given for the details.

[8] 7. There were 184 students registered in CPSC 121 in term 2 of last year. Using an indirect proof, prove that there were two CPSC 121 students who either had the same birthday, or birthdays that occur on 2 consecutive days.