CPSC 121,
2007/8 Winter Term 2
Section 203: Quiz 3

Name: _____ Student ID: _____

Signature: _____

- You have **20 minutes** to write the **3 questions** on this examination.
- A total of 18 marks are available. **You may want to complete what you consider to be the easiest questions first!**
- Justify all of your answers.
- No notes or electronic equipment are allowed.
- Keep your answers short. If you run out of space for a question, you have written too much.
- The number in square brackets to the right of the question indicates the number of marks allocated to that question.
- Good luck!

Question	Marks	Out of
1		6
2		6
3		6
Total		18

UNIVERSITY REGULATIONS:

- Each candidate should be prepared to produce, upon request, his/her university-issued ID.
- No candidate shall be permitted to enter the examination room after the expiration of one half hour or to leave during the first half hour of the examination.
- CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
    1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
    2. Speaking or communicating with other candidates.
    3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

1. **[6]** 1. Your friend is writing a regular expression to search the classifieds for a car to buy. She wants to match ads that contain the words “car”, “for”, “sale” (in the order) followed by a price between $2000 and $2999 obo. So far, she has the regular expression:

   ```
   .*car.*for.*sale.*\$2[0-9]{3}\sobo
   ```

   Unfortunately, this produces some errors on her test suite:

   Strings that should be accepted:
   car for sale in Coquitlam, 2500 obo (rejected)
   Silver car for sale, $2750 obo (accepted)
   Brand new car for sale, $2999 obo (accepted)

   Strings that should be rejected:
   truck for sale, $2500 (rejected)
   cartier watch for sale, $2250 obo (accepted)
   for sale: car - $2100 (rejected)

   Her regular expression rejects “car for sale in Coquitlam, 2500 obo” (which should be accepted) and accepts “cartier watch for sale, $2250 obo” (which should be rejected). Correct the errors in the regular expression. Your answer should match the intended strings and, in particular, should correctly accept or reject all of the strings in the test suite.

[6] 2. Prove the following for arbitrary sets `P`, `Q`, `R`, and `S`: $(P \cup \overline{R}) \cap (Q \cup R) \subseteq P \cup Q \cup S$. Your proof may be in a combination of English and logic but make sure that it's clear what the steps of your proof are and how each step follows from the previous steps. (Your proof may not take the form of a Venn Diagram.)

[6] 3. Consider the set A = {hello, 3, ∅}, its power set P(A), and its Cartesian product with itself A × A. For each of the following functions, give an example value from the domain and its corresponding image. Then, say whether the function is (**Y**) or is not (**N**) injective and/or surjective. The first row is filled in as an example.

The function `f`	Pre-image	Image	Injective? (**Y** or **N**)	Surjective? (**Y** or **N**)
`f : A` → `A` maps values onto themselves: `f(x) = x`.	`hello`	`hello`	Y	Y
`f : A` ∩ `A` → `A` ∪ `A` maps values onto themselves: `f(x) = x`.				
`f : A × A × A` → `P(A) − ∅` turns tuples into sets: `f(x,y,z) = {x,y,z}`.				
`f : A` → `A` depends on cardinality: `f(x) =` $\begin{cases} 3 & \text{if } \lvert A\times A\rvert > \lvert P(A)\rvert \\ \text{hello} & \text{if } \lvert A\times A\rvert = \lvert P(A)\rvert \\ \{\} & \text{if } \lvert A\times A\rvert < \lvert P(A)\rvert \end{cases}$				

Appendix: Regular Expression Reference

Character(s)	Description	Example
`.`	Matches any character	`.at ={"aat", "bat", ...}`
`[ ]`	Matches one character from those listed	`[cbr]at = {"cat", "bat", "rat"}`
`[ - ]`	Matches one character from the range of characters listed	`[a-c]at = {"bat", "cat", "dat"}`
`[^ ]`	Matches one character from those not listed	`[^b-d]at = {"aat","eat", "fat",...}`
`|`	Matches one item from those separated by pipes |	`(ph|meerk|r)at = {"phat","meerkat","rat"}`
`*`	Repeat the previous item 0 or many times	`ca*t = {"ct", "cat", "caat", ...}`
`+`	Repeat the previous item 1 or many times	`ca+t = {"cat", "caat", ...}`
`?`	Repeat the previous item 0 or 1 time	`ca?t = {"ct","cat"}`
`{#}`	Repeat the previous item an exact number of times	`ca{3}t = {"caaat"}`
`{#,}`	Repeat the previous item a min. number of times	`ca{2,}t = {"caat","caaat",...}`
`{#,#}`	Repeat the previous item between a min. and max. number of times	`ca{1,3}t = {"cat","caat","caaat"}`

In addition, there are a few special character codes that come in handy:

Symbol	Description
`\d`	Equivalent to `[0-9]`: Matches any digit
`\D`	Equivalent to `[^0-9]`: Matches any non-digit
`\s`	Matches white space character
`\S`	Matches a non-white space character
`\w`	Equivalent to `[a-zA-Z0-9_]`: Matches a "word" character
`\W`	Equivalent to `[^a-zA-Z0-9_]`: Matches a non-word character

Law Name	Equivalences
Identity	$p \wedge T \equiv p$ $p \vee F \equiv p$
Domination	$p \wedge F \equiv F$ $P \vee T \equiv T$
Distributive	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Absorption	$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$
Negation	$p \wedge \sim p \equiv F$ $p \vee \sim p \equiv T$
De Morgan's	$\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$ $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$
Idempotent	$p \wedge p \equiv p$ $p \vee p \equiv p$
Commutative	$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$
Associative	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ $(p \vee q) \vee r \equiv p \vee (q \vee r)$
Definition of conditional	$p \rightarrow q \equiv \sim p \vee q$
Contrapositive	$p \rightarrow q \equiv \sim q \rightarrow \sim p$

Set Identity	Equivalences
Identity	$A \cap \mathcal{U} \equiv A$ $A \cup \varnothing \equiv A$
Domination	$A \cap \varnothing \equiv \varnothing$ $A \cup \mathcal{U} \equiv \mathcal{U}$
Distributive	$A \cap (B \cup C) \equiv (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) \equiv (A \cup B) \cap (A \cup C)$
Absorption	$A \cup (A \cap B) \equiv A$ $A \cap (A \cup B) \equiv A$
Complement	$A \cap \overline{A} \equiv \varnothing$ $A \cup \overline{A} \equiv \mathcal{U}$
De Morgan's	$\overline{(A \cap B)} \equiv \overline{A} \cup \overline{B}$ $\overline{(A \cup B)} \equiv \overline{A} \cap \overline{B}$
Idempotent	$A \cap A \equiv A$ $A \cup A \equiv A$
Commutative	$A \cap B \equiv B \cap A$ $A \cup B \equiv B \cup A$
Associative	$A \cap (B \cap C) \equiv (A \cap B) \cap C$ $A \cup (B \cup C) \equiv (A \cup B) \cup C$
Complementation	$\overline{\overline{A}} \equiv A$
Set Difference	$A - B = A \cap \overline{B}$