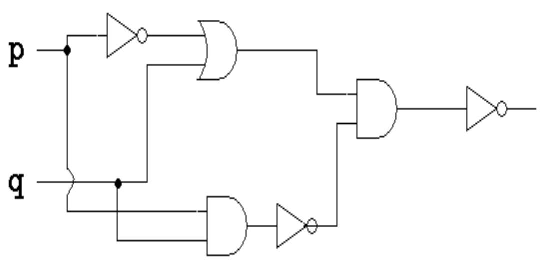
CPSC 121 Midterm Examination
July 9th, 2007

[10] 1. Match each proposition in the left column with the logically equivalent proposition in the right column. Each proposition from the left column is equivalent to only one proposition in the right column. Since there are two propositions in the left column and five propositions in the right column, three of the propositions in the right column will not be used when you make your matches. Justify your answers (i.e. for each pair, show why the two propositions are logically equivalent).

a. $(\sim p \vee q) \wedge (\sim q \vee p)$

b.



1. $p \vee q$
2. p if and only if q
3. q
4. p only if q
5. p

Solution:

- a matches with 2
- b matches with 5

valid justifications are truth tables or showing that the two propositions are logically equivalent using logical equivalences

- a with 2

$(\sim p \vee q) \wedge (\sim q \vee p)$	
$(p \rightarrow q) \wedge (q \rightarrow p)$	Definition of$\rightarrow$
$(p \leftrightarrow q)$	Definition of$\leftrightarrow$

- b with 5

  First, we need to translate from the circuit to propositional logic.

  The circuit is equivalent to $\sim((\sim p \vee q) \wedge \sim(q \wedge p))$

$\sim((\sim p \vee q) \wedge \sim(q \wedge p))$	
$\sim(\sim p \vee q) \vee (q \wedge p)$	De Morgan's
$(p \wedge \sim q) \vee (q \wedge p)$	De Morgan's
$p \wedge (\sim q \vee q)$	Distributive Law
$p \wedge T$	Negation Law
p	Identity Law

[10] 2. Prove or disprove that the following argument is valid. Please clearly state the rule of inference you are using for each step.

1. $m \wedge n$
2. $\sim(n \wedge j)$
3. $r \rightarrow n$
4. $k \rightarrow j$
5. $\sim k \rightarrow \sim(s \vee \sim p)$

$\therefore p$

Solution: This argument is valid.

6.	n	Simplification - 1
7.	$\sim n \vee \sim j$	De Morgan's - 2
8.	$\sim j$	Disjunctive Syllogism - 6, 7
9.	$\sim k$	Modus Tollens - 4, 8
10.	$\sim(s \vee \sim p)$	Modus Ponens - 9, 5
11.	$\sim s \wedge p$	De Morgan's - 10
$\therefore p$		Simplification - 11

[8] 3. Translate each of the following English propositions into predicate logic. Assume that P is the set of all people and A is the set of all animals. You can define any predicates that you need, but you cannot define any new sets - you must use P or A.

[4] a. Every dog has an owner.

Solution: Let $D(x)$ mean "x is a dog", and O(x,y) mean "x owns y". Then, the proposition can be written as:
$\forall x \in A, D(x) \rightarrow \exists y \in P, O(y, x)$

[4] b. If some lion is the king of the desert, then some gorilla is the queen of the jungle.

Solution: Let $L(x)$ mean "x is a lion", $G(x)$ mean "x is a gorilla", $K(x)$ mean "x is the king of the desert", and $Q(x)$ mean "x is the queen of the jungle". Then, the proposition can be written as:
$(\exists x \in A, L(x) \wedge K(x)) \rightarrow (\exists y in A, G(y) \wedge Q(y))$

[10] 4. [2] a. Translate $001D_{16}$ to decimal.

Solution: 29_{10}

[6] b. For the following 6 bit binary integers, give the equivalent decimal value if it is interpreted as:
1) an unsigned integer
2) a signed integer in sign bit representation
3) a signed integer in two's complement representation

110010

Solution : • unsigned integer: 50 (32 + 16 + 2)

- sign bit: -18 (negative because of the 1 in the left-most bit, magnitude = 16 + 2)
- two's complement: -14 (-32 + 16 + 2)

011100

Solution : • unsigned integer: 28 (16 + 8 + 4)

- sign bit: 28 (positive because of the 0 in the left-most bit, magnitude = 16 + 8 + 4)
- two's complement: 28 (16 + 8 + 4)

[2] c. We discussed base 2, base 10 and base 16 in class. What is the decimal equivalent of 1023_4 (i.e. 1023 in base 4)?

Solution : 75 = $(3 * 4^0) + (2 * 4^1) + (0 * 4^2) + (1 * 4^3) = 3 + 8 + 0 + 64$

[10] 5. Prove that for all integers m, n, and d, if $(m \text{ mod } d) = (n \text{ mod } d)$, then $d|(m - n)$.

Remember from class that if $p|q$, then there exists some integer x such that $px = q$.

Definition of mod

x mod y returns the remainder after x is divided by y. So, if $x = zy + r$ (for some z), x mod y = r. Here are some examples:

- 15 mod 4 = 3
- 19 mod 4 = 3
- 21 mod 2 = 1
- 3 mod 3 = 0
- 4 mod 3 = 1

[12] 6. Consider each of the following statements related to divisibility and prime numbers. In each case, either prove that the statement is true (using an indirect proof) or prove the statement is false by giving a counterexample:

Hint: Remember that you can use the theorems we proved in class as lemmas and that a number is prime if it's only factors are 1 and itself.

[6] a. If n is a prime number that is strictly greater than 2, then n^2 is odd.

Solution : This statement is true. We shall prove the contrapositive. That is, we show that if n is an integer greater than 2, and n^2 is even, then n is not prime. So, consider an integer n greater than 2, whose square is even. We proved in class in section 101 (and it is also Example 24 in section 1.5 of Rosen, page 681) that this implies that n is even. Therefore $n = 2x$. Since $n > 2$, it follows that 2 is a divisor of n other than n and 1. In other words, n is not a prime number. □

[6] b. If $n = p^2 + q^2$, where p and q are distinct primes (i.e. $p \neq q$), then n is also a prime.

Solution : This is false. To prove the falsehood of the statement, all we need to do is find one counterexample. Take $p = 5$ and $q = 7$. Then $p^2 + q^2 = 25 + 49 = 74$, which is not a prime (see the proof in part (a)).