CPSC 121 Midterm Examination
July 9th, 2007

Name: ____________________ Student ID: ____________________
Signature: ____________________

- You have 60 minutes to write the 6 questions on this examination. A total of 60 marks are available.
- **Justify all of your answers.**
- You are allowed to bring in one double-sided 8.5 x 11 sheet of notes (handwritten or typed using at least size 12 point font), and nothing else.
- Keep your answers short. If you run out of space for a question, you have written too much.
- The number in square brackets to the left of the question number indicates the number of marks allocated for that question. Use these to help you determine how much time you should spend on each question.
- Use the back of the pages for your rough work.
- **Good luck!**

Question	Marks
1	
2	
3	
4	
5	
6	
Total	

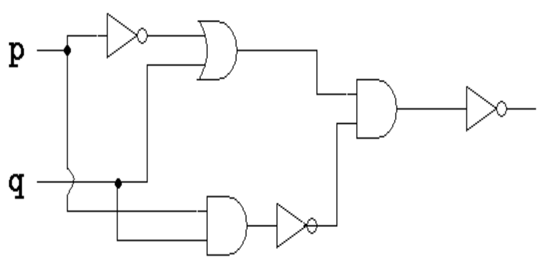
UNIVERSITY REGULATIONS:

- Each candidate should be prepared to produce, upon request, his/her library card.
- No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.
- CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
  2. Speaking or communicating with other candidates.
  3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

[10] 1. Match each proposition in the left column with the logically equivalent proposition in the right column. Each proposition from the left column is equivalent to only one proposition in the right column. Since there are two propositions in the left column and five propositions in the right column, three of the propositions in the right column will not be used when you make your matches. Justify your answers (i.e. for each pair, show why the two propositions are logically equivalent).

a. $(\sim p \vee q) \wedge (\sim q \vee p)$

b.



1. $p \vee q$
2. p if and only if q
3. q
4. p only if q
5. p

[10] 2. Prove or disprove that the following argument is valid. Please clearly state the rule of inference you are using for each step.

1. $m \wedge n$
2. $\sim(n \wedge j)$
3. $r \rightarrow n$
4. $k \rightarrow j$
5. $\sim k \rightarrow \sim(s \vee \sim p)$

$\therefore p$

[8] 3. Translate each of the following English propositions into predicate logic. Assume that P is the set of all people and A is the set of all animals. You can define any predicates that you need, but you cannot define any new sets - you must use P or A.

[4] a. Every dog has an owner.

[4] b. If some lion is the king of the desert, then some gorilla is the queen of the jungle.

[10] 4. [2] a. Translate $001D_{16}$ to decimal.

[6] b. For the following 6 bit binary integers, give the equivalent decimal value if it is interpreted as:
1) an unsigned integer
2) a signed integer in sign bit representation
3) a signed integer in two's complement representation

110010

011100

[2] c. We discussed base 2, base 10 and base 16 in class. What is the decimal equivalent of 1023_4 (i.e. 1023 in base 4)?

[10] 5. Prove that for all integers m, n, and d, if $(m \text{ mod } d) = (n \text{ mod } d)$, then $d|(m - n)$.

Remember from class that if $p|q$, then there exists some integer x such that $px = q$.

Definition of mod

x mod y returns the remainder after x is divided by y. So, if $x = zy + r$ (for some z), x mod y = r. Here are some examples:

- 15 mod 4 = 3
- 19 mod 4 = 3
- 21 mod 2 = 1
- 3 mod 3 = 0
- 4 mod 3 = 1

[12] 6. Consider each of the following statements related to divisibility and prime numbers. In each case, either prove that the statement is true (using an indirect proof) or prove the statement is false by giving a counterexample:

Hint: Remember that you can use the theorems we proved in class as lemmas and that a number is prime if it's only factors are 1 and itself.

[6] a. If n is a prime number that is strictly greater than 2, then n^2 is odd.

[6] b. If $n = p^2 + q^2$, where p and q are distinct primes (i.e. $p \neq q$), then n is also a prime.