CPSC 121 Quiz 4
November 22nd, 2007

[4] 1. What set of strings does the regular expression `\(\d+(,\d*\d)*\)` describe? Justify your answer briefly.

Recall that `\d` represents a single digit, and that `*` and `+` respectively mean 0 or more, and 1 or more copies of the preceding regular expression (the comma character "`,`" has no special meaning).

Solution : This regular expressions matches comma-separated lists of 1 or more non-negative integers, enclosed in parentheses.

[8] 2. Use mathematical induction to prove that for every $n \geq 1$, $1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + 4 \cdot 6 + \cdots + n(n+2) = n(n+1)(2n+7)/6$. Part of your grade will depend on how your proof is constructed; the rest will be for the details.

Solution : First, we prove the base case, that is the case $n = 1$. Since $1 \cdot 3 = 3$, and $1 \cdot 2 \cdot (2 \cdot 1 + 7)/6 = 1 \cdot 2 \cdot 9/6 = 3$, the theorem holds for $n = 1$.

Now, we prove the induction step. Pick an unspecified integer $n \geq 1$, and suppose that

$$\sum_{i=1}^{n} i \cdot (i+2) = \frac{n(n+1)(2n+7)}{6}.$$

We now need to show that the same thing holds for $n + 1$. Indeed,

$$\begin{aligned}
\sum_{i=1}^{n+1} i \cdot (i+2) &= \left(\sum_{i=1}^{n} i \cdot (i+2)\right) + (n+1)(n+3) \\
&= \frac{n(n+1)(2n+7)}{6} + (n+1)(n+3) \\
&= \frac{n(n+1)(2n+7)}{6} + \frac{6(n+1)(n+3)}{6} \\
&= \frac{n+1}{6}(n(2n+7) + 6(n+3)) \\
&= \frac{n+1}{6}(2n^2 + 13n + 18) \\
&= \frac{n+1}{6}(2n+9)(n+2) \\
&= \frac{(n+1)(n+2)(2(n+1)+7)}{6}
\end{aligned}$$

This completes the induction step. Hence, by the principle of mathematical induction, the theorem holds.

[8] 3. Design a DFA that reads in a sequence of bits, and accepts the sequence if and only if the 1's occur only in groups of 2. For instance, your DFA should accept the sequences "", "000", "0011" and "1101100", but not "1", "011110" or "0100110". Hint: there is a solution with only 4 states. Draw your DFA on the back of the page.

Solution :