CPSC 121 Quiz 3
November 8^{th}, 2007

[9] 1. Let A and B be the two sets defined by

$$\begin{aligned} A &= \{v \in \mathbf{Z}^+ \mid \exists w \in \mathbf{Z}^+ v = 4w - 1\} \\ B &= \{x \in \mathbf{Z}^+ \mid \exists y \in \mathbf{Z}^+ x = 4y + 3\} \end{aligned}$$

[7] (a) Prove that $B \subseteq A$.

Solution: We need to show that for every $x \in B$, $x \in A$. So pick an unspecified element x of B. There is a positive integer y such that $x = 4y + 3$. Since $4y + 3 = 4(y + 1) - 1$, this means that $x = 4(y + 1) - 1$. Since clearly $y + 1$ is a positive integer, it follows that $x \in A$.

[2] (b) Are A and B equal? Why or why not?

Solution: No, they are not: $3 \in A$ since $3 = 4 \cdot 1 - 1$, but $2 \notin B$ (while $3 = 4 \cdot 0 + 3$, 0 is not a positive integer).

[11] 2. Consider two unspecified functions $f : B \to C$ and $g : A \to B$.

[7] a. Prove that if g is not one-to-one, then $f \circ g$ is not one-to-one either (it doesn't matter whether or not f is one-to-one).

Solution: Suppose that g is not one-to-one. This means that there are two distinct elements x and y of A such that $g(x) = g(y)$. Then, clearly, $f(g(x)) = f(g(y))$, which means that $f \circ g$ is not one-to-one.

[4] b. Now, give an example that shows that if g is not onto, it is still possible for $f \circ g$ to be onto.

Solution: Let $A = \{1\}$, $B = \{1, 2\}$, $C = \{1\}$, $g : A \to B$ be defined by $g(1) = 1$, and $f : B \to C$ be defined by $f(1) = 1, f(2) = 1$. Clearly g is not onto, but $f \circ g$ is.