CPSC 121 Quiz 2
October 11^{th}, 2007

[9] 1. Translate each of the following English propositions into predicate logic. Assume that P is the set of well-known painters, C is a set of countries, S is the set of all current UBC students, *Likes*(x, y) is a predicate that is true if student x likes paintings by painter y, and *EatsAt*(x, y) is a predicate that is true if student x eats at restaurant selling food typical of country y. Assume that a student only eats at a restaurant if he/she enjoys the food cooked there, and only buys paintings he/she likes.

[3] a. One student bought a painting by Picasso immediately after eating at McDonald's (note: hamburgers are american food).

Solution : $\exists s \in S$ *Likes*$(s, \text{Picasso}) \wedge$ *EatsAt*(s, USA)

[3] b. Only students who like paintings by Salvador Dali will eat in Greek restaurants.

Solution : $\forall s \in S$ *EatsAt*$(s, \text{Greece}) \rightarrow$ *Likes*(s, Dali)

[3] c. Students who dislike both Monet and French food do not like Renoir either.

Solution : $\forall s \in S \ \sim$*Likes*$(s, \text{Monet}) \wedge \sim$*EatsAt*$(s, \text{France}) \rightarrow \sim$*Likes*$(s, \text{Renoir})$

[5] 2. Using a direct proof, show that if an integer a divides an integer b, and that b divides an integer c, then a divides c.

Solution : Consider unspecified integers a, b and c. Assume that a divides b, and that b divides c. Since a divides b, we know that $b = ax$ for some integer x. Since b divides c, we know that $c = by$ for some integer y. Hence

$$c = by = (ax)y = a(xy)$$

and since xy as a product of two integers is also an integer, this means that a divides c.

[6] 3. Let A be a set. Explain how you would prove a statement of the form $\forall x \in A, \ \exists y \in A, \ \forall z \in A, \ P(x, y, z) \rightarrow Q(x, y, z)$ using a direct proof. That is, describe the outline of the proof (you obviously can not provide details since I have not told you anything about A, P and Q).

Solution : You would write the proof as follows:

```
Consider an unspecified element x or A
  Choose y to be (something that might depend on x)
    Consider an unspecified element z of A
      Assume P(x, y, z) is true
      Then somehow prove that Q(x, y, z) holds.
```

Telling us how to choose y and prove that $Q(x, y, z)$ holds would require knowing what $P(x, y, z)$ and $Q(x, y, z)$ stand for, and what A is.