CPSC 121 Quiz 1
September 25^{th}, 2007

[12] 1. Propositional Logic and Circuits

[3] a. Using a sequence of known logical equivalences (not a truth table), prove that $\rightarrow$ distributes over $\wedge$. That is, prove that $p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$.

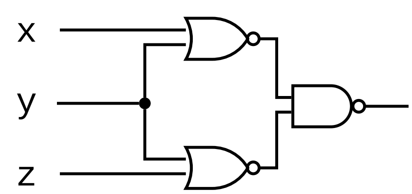
Solution:

$$\begin{aligned} p \rightarrow (q \wedge r) &\equiv (\sim p) \vee (q \wedge r) \\ &\equiv (\sim p \vee q) \wedge (\sim p \vee r) \\ &\equiv (p \rightarrow q) \wedge (p \rightarrow r) \end{aligned}$$

[3] b. Does $\wedge$ distribute over $\rightarrow$? That is, is $p \wedge (q \rightarrow r)$ logically equivalent to $(p \wedge q) \rightarrow (p \wedge r)$? Explain why or why not.

Solution: No, $\wedge$ does not distribute over $\rightarrow$. If p is false, then $p \wedge (q \rightarrow r)$ is also false. However when p is false, $(p \wedge q)$ is false, which means that $(p \wedge q) \rightarrow (p \wedge r)$ is true. Therefore $p \wedge (q \rightarrow r)$ is not logically equivalent to $(p \wedge q) \rightarrow (p \wedge r)$.

[6] c. Prove that the output of the following circuit is logically equivalent to $x \vee y \vee z$.



Solution: The output from the circuit is $(x \text{ nor } y)$ nand $(y \text{ nor } z)$. Moreover

$$\begin{aligned} (x \text{ nor } y) \text{ nand } (y \text{ nor } z) &\equiv \sim((x \text{ nor } y) \wedge (y \text{ nor } z)) \\ &\equiv \sim(x \text{ nor } y) \vee \sim(y \text{ nor } z) \\ &\equiv \sim\sim(x \vee y) \vee \sim\sim(y \vee z) \\ &\equiv (x \vee y) \vee (y \vee z) \\ &\equiv ((x \vee y) \vee y) \vee z \\ &\equiv (x \vee (y \vee y)) \vee z \\ &\equiv (x \vee y) \vee z \\ &\equiv x \vee y \vee z \end{aligned}$$

[8] 2. Determine the validity of the following argument. Fully justify your answer.

1. $t \to \sim p$
2. $s \to \sim(q \wedge r)$
3. $(p \wedge u) \to v$
4. $p \wedge q \wedge r$
5. $s \vee t \vee u$

$\therefore v$

Solution: The argument is valid.

	Step	**Reason**
6.	$q \wedge r$	Simplification from (4)
7.	$\sim\sim(q \wedge r)$	Double-negation law from (6)
8.	$\sim s$	Modus tollens from (2),(7)
9.	$t \vee u$	Disjunctive syllogism from (5), (8)
10.	p	Simplification from (4)
11.	$\sim\sim p$	Double-negation law from (10)
12.	$\sim t$	Modus tollens from (1),(11)
13.	u	Disjunctive syllogism from (5), (12)
14.	$p \wedge u$	From (10) and (13)
15.	v	Modus ponens from (3), (14)